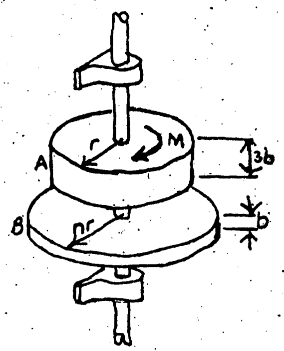
Two disks of the same material are attached to a shaft as shown. Disk A is of radius r and has a thickness 3b, while disk B is of radius nr and thickness b. A couple M of constant magnitude is applied when the system is at rest and is removed after the system has executed two revolutions. Determine the value of n which results in the largest final speed for a point on the rim of disk B.



For a disk $I = \frac{1}{2} mR^2 = \frac{1}{2}(\rho \pi R^2 t)R^2$

$$= \frac{1}{2}\pi\rho\, tR^4$$

Where ρ = mass density of material

t = thickness

R = radius

$$I_B = I_A \frac{(nR)^4}{R^4} \frac{b}{3b} = \frac{n^4}{3} I_A$$

Thus $I_{Total} = I_A + I_B = I_A(1 + \frac{n^4}{3})$ (1)

Principle of Work-Energy

$$\cancel{E_1}^{\,0} + W_2 = E_2 + \cancel{{}_1V_2}^{\,0}$$

$$M(4\pi) = \frac{1}{2} I_A(1 + \frac{n^4}{3}) W_2^2$$

$$W_2^2 = \frac{8\pi M}{I_A} \frac{1}{1+\frac{n^4}{3}} = \frac{24\pi M}{I_A} \frac{1}{3 + n^4}$$

For a point D on the Rim of Disk B

$$V_D = (nr)W_2 \qquad V_D^{\,2} = n^2r^2W_2^{\,2} = \frac{24\pi r^2 M}{I_A} \times \frac{n^2}{3 + n^4}$$

Max V_D occurs for max. $(\frac{n^2}{3 + n^4})$

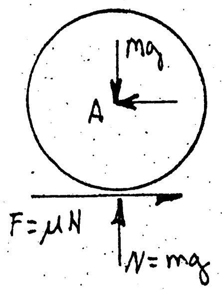
$$\frac{\partial}{\partial n} (\frac{n^2}{3 + n^4}) = \frac{1}{(3 + n^4)^2} \left[(3 + n^4)(2n) - n^2(4n^3)\right]$$

$$[\ ] = 0; \quad 2n\left[3 + n^4 - 2n^4\right] = 2n\left[3 - n^4\right]$$

$$n = 0 \quad \text{and} \quad n^4 = 3$$

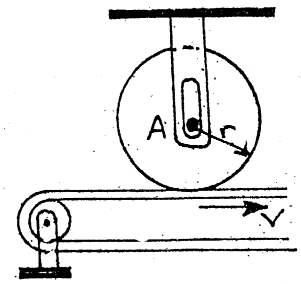
$$n = \sqrt[4]{3} = 1.316$$

A disk of constant thickness and initially at rest is placed in contact with the belt, which moves with a constant velocity v. Denoting by μ the coefficient of friction between the disk and the belt, derive an expression for the number of revolutions executed by the disk before it reaches a constant angular velocity.



The only force doing work is F. Since it's moment about A is M = rF, we have

$${}_1W_2 = M\theta = Fr\theta = \mu\, mgr\theta$$



$$\cancel{E_1}^{\,0} + {}_1W_2 = E_2 + \cancel{{}_1V_2}^{\,0}$$

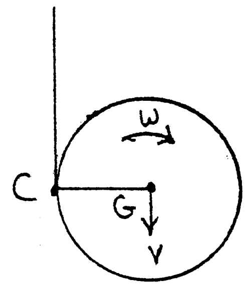
$$E_2 = \frac{1}{2} IW^2 = \frac{1}{2}\left(\frac{1}{2} mr^2\right)\left(\frac{V}{r}\right)^2$$

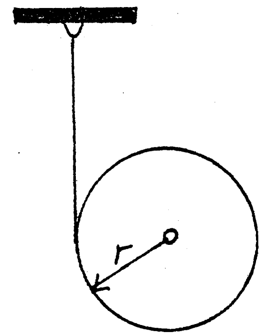
$$\mu mgr\theta = \frac{1}{2}\left(\frac{1}{2} mr^2\right)\left(\frac{V}{r}\right)^2$$

$$\mu mgr\theta = \frac{1}{4} mV^2$$

$$\theta = \frac{V^2}{4r\mu g} \text{ radians} \qquad \theta = \frac{V^2}{8\pi r \mu g} \text{ revolutions}$$

A cord is wrapped around a cylinder of radius r and mass m as shown. If the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved through a distance h.





KINEMATICS Since $V_C = 0$, we have $V = rw$ $\quad w = \frac{V}{r}$

$$\cancelto{0}{E_1} + {}_1W_2 = E_2 + \cancelto{0}{{}_1V_2} \qquad {}_1W_2 = mgh$$

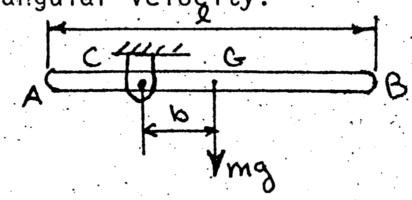
$$mgh = \frac{1}{2} mV^2 + \frac{1}{2} Iw^2$$

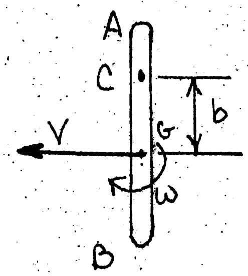
$$= \frac{1}{2} mV^2 + \frac{1}{2}\left[\frac{1}{2} mr^2\right]\left(\frac{V}{r}\right)^2$$

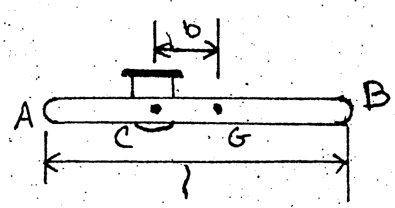
$$= \frac{3}{4} mV_2^{\,2}$$

$$V_2 = \sqrt{\frac{4gh}{3}} \downarrow$$

A uniform rod of length l is pivoted about a point C located at a distance b from its center G. The rod is released from rest in a horizontal position. Determine (a) the distance b so that the angular velocity of the rod as it passes through a vertical position is maximum, (b) the value of the maximum angular velocity.







Rotation about C: $V = bw$

$E_1 = 0 \qquad {}_1W_2 = mgh \qquad E_2 = \frac{1}{2} mV^2 + \frac{1}{2} Iw^2 = \frac{1}{2} m(bw)^2 + \frac{1}{2}\left[\frac{1}{12} m \ell^2\right] w^2$

$$E_2 = \frac{1}{2} m \left[b^2 + \frac{1}{12}\ell^2\right] w^2$$

$$E_1 + {}_1W_2 = E_2$$

$$mgb = \frac{1}{2} m \left[b^2 + \frac{1}{12}\ell^2\right] w^2$$

$$w^2 = 2g\left[\frac{b}{b^2 + \frac{1}{12}\ell^2}\right] \qquad (1)$$

a) <u>MAXIMUM W OCCURS WHEN W^2 IS MAXIMUM</u>

$$\frac{d(w^2)}{db} = \frac{2g}{\left[b^2 + \frac{1}{12}\ell^2\right]^2}\left[(b^2 + \frac{1}{12}\ell^2) - b(2b)\right] = 0$$

$$\left[b^2 + \frac{1}{12}\ell^2\right] = 0 \qquad b = \frac{\ell}{\sqrt{12}}$$

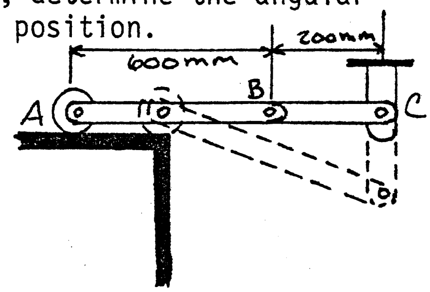
b) <u>VALUE OF MAXIMUM W</u> Use Eqn. (1) with $b = \frac{\ell}{\sqrt{12}}$

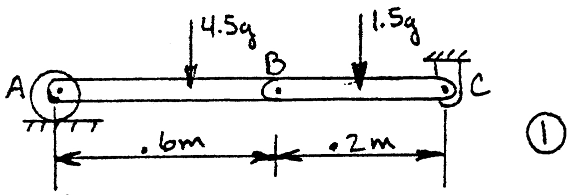
$$w^2 = 2g\,\frac{\ell/\sqrt{12}}{(\ell^2/12) + (\ell^2/12)} = 2g\,\frac{\ell/\sqrt{12}}{2\ell^2/12} = \sqrt{12}\,\frac{g}{\ell}$$

$$w = \sqrt[4]{12}\,\sqrt{g/\ell}$$

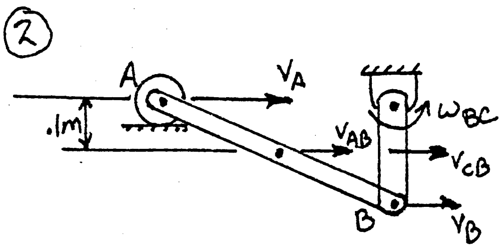
$$w = 1.861\,\sqrt{g/\ell}$$

The uniform rods AB and BC are of mass 4.5 and 1.5 kg respectively. If the system is released from rest in the position shown, determine the angular velocity of rod BC as it passes through a vertical position.





$E_1 = 0$



In position 2, we note that V_A and V_B are parallel, ∴ AB is in translation, $W_{AB} = 0$

$$V_{AB} = V_A = V_B = .2W_{BC}$$

$$E_2 = \frac{1}{2} M_{AB} V_{AB}^2 + \left[\frac{1}{2} M_{BC} V_{BC}^2 + \frac{1}{2} I_{BC} W_{BC}^2\right]$$

$$= \frac{1}{2}(4.5)(.2W_{BC})^2 + \frac{1}{2}(1.5)(.1W_{BC})^2 + \frac{1}{2}\left[\frac{1}{12}(1.5)(.2)^2\right] W_{BC}^2$$

$$E_2 = 0.1\, W^2_{BC}$$

$$_1L_2 = -(4.5\text{kg})(.1\text{m}) - 1.5\text{kg}(.1\text{m}) = 6(9.81)(.1)$$
$$= -5.89\text{j}$$

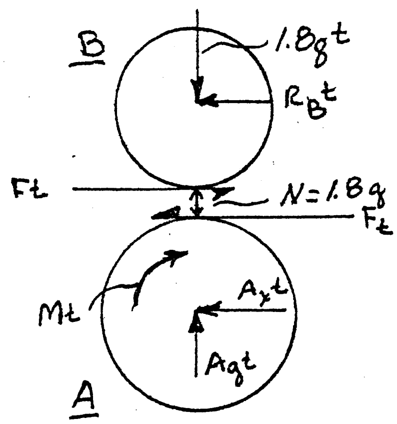
Conservation of Energy

$$E_1 + {_1W_2} = E_2 + {_1L_2}$$

$$0 + 0 = .1\, W_{BC}^2 - 5.89$$

$$W_{BC} = 7.67 \text{ rad/sec}$$

Disks A and B are of mass 5 and 1.8kg respectively. The disks are initially at rest and the coefficient of friction between them is .20. A couple M of magnitude 4N-m is applied to disk A for 1.50s and then removed. Determine (a) whether slipping occurs between the disks, (b) the final angular velocity of each disk.

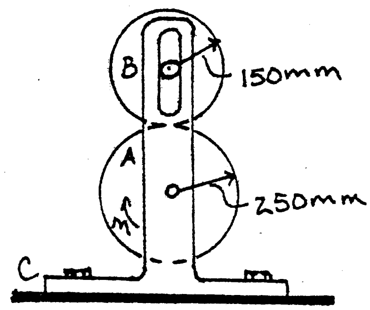


Assume slipping does not occur:

$$r_A w_A = r_B w_B$$

$$.25\ w_A = .15\ w_B$$

$$w_A = .6\ w_B \qquad (1)$$



IMPULSE:

Disk B: ↻ +

$$Ftr_B = I_B w_B \qquad Ft(.15) = \frac{1}{2}(1.8)(.15)^2 w_B \qquad (2)$$

Disk A: + ↻

$$Mt - Ftr_A = I_A w_A$$

$$Mt - Ft(.25) = \frac{1}{2}(5)(.25)^2 w_A \qquad (3)$$

Mult. (3) by .6 and add to EQ. (2) to eliminate 'Ft' term

$.6Mt = .09375w_A + .02025w_B$ Subst w_A from EQ (1)

And w_B = 47.06 rad/sec

Subst. w_B into EQN (2) and find 'F'

$F = 4.24$

$F = \mu N \qquad \mu = \frac{F}{N} = \frac{4.24}{1.8g} = .24 > \mu_{Given}$

(a) ∴Slipping occurs

(b) $F = .2(1.8g) = 3.53$ → Subst. into EQ's (2) and (3)

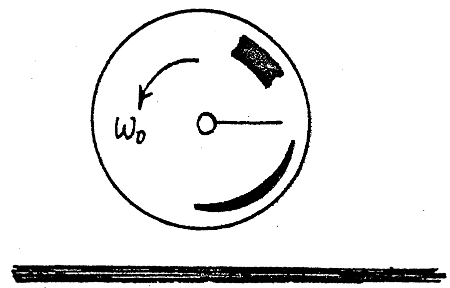
From EQN (2)

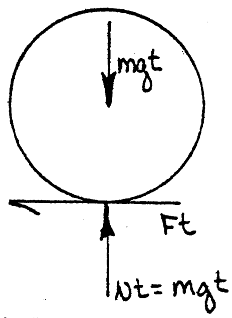
w_B = 39.2 rad/sec ↺

From EQN (3)

w_A = 29.9 rad/sec ↻

A sphere of mass m and radius r is projected along a rough horizontal surface with the initial velocities indicated. If the final velocity of the sphere is to be zero, express (a) the required omega$_0$ in terms of $\bar{v}_0$ and r, (b) the time required for the sphere to come to rest in terms of $\bar{v}_0$ and μ.





Initial momentum and impulse equals final momentum

$\xrightarrow{+}$ x component: $mV_0 - Ft = 0$ (1)

$\overset{+}{\circlearrowleft}$ Moments about G: $Iw_0 - (Ft)r = 0$ (2)

(a) Eliminating 'Ft'

$$Iw_0 - (mV_0)r = 0$$

$$I = \frac{2}{5} mr^2 \qquad \frac{2}{5} mr^2 w_0 = mV_0 r$$

$$w_0 = \frac{5}{2}\frac{V_0}{r} \circlearrowleft$$

(b) We have: $Ft = \mu Nt = \mu mgt$

$F = \mu mg$

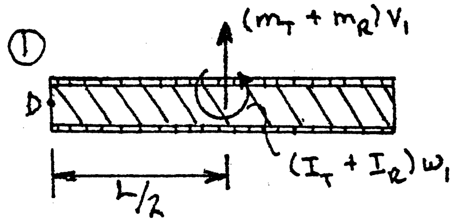
From EQ. (1): $$t = \frac{mV_0}{F} = \frac{mV_0}{\mu mg}$$

$$t = \frac{V_0}{\mu g}$$

DYNAMICS

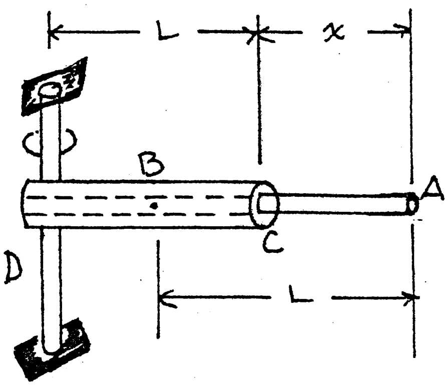
PROBLEM N9/9

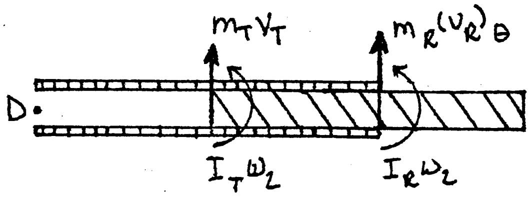
The rod AB is of mass m and slides freely inside the tube CD which is also of mass m. The angular velocity of the accembly was omega$_1$ when the rod was entirely inside the tube (x = 0). Neglecting the effect of friction, determine the angular velocity of the assembly when x = L/2.



T - tube

R - rod





Top views

Moments about shaft at D

Momentum conserved

$$(m_T + m_R)V_1 \frac{L}{2} + (I_T + I_R)w_1 = m_T V_T(\frac{L}{2}) + m_R(V_R)_\theta L + (I_T + I_R)w_2 \qquad (1)$$

Since both rod and tube are of length L and Mass m;

$$m_T = m_R = m \qquad I_T = I_R = \frac{1}{12} mL^2$$

$$V_1 = \frac{L}{2}w \qquad V_T = \frac{L}{2}w_2 \quad (V_R)_\theta = Lw_2$$

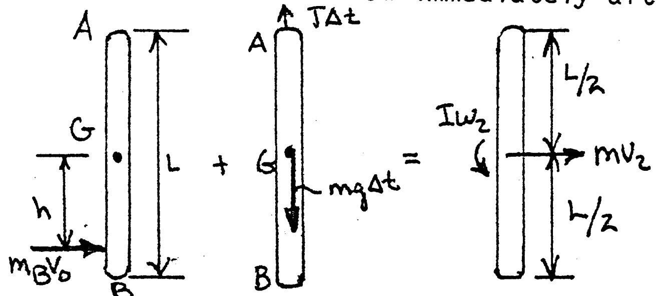
Substitute above items into EQ(1); we get

$$2m(\frac{L}{2} w_1)\frac{L}{2} + 2(\frac{mL^2}{12})w_1 = m(\frac{L}{2}w_2)\frac{L}{2} + m(Lw_2) + 2(\frac{mL^2}{12}) w_2$$

$$\frac{2}{3} mL^2 w_1 = \frac{17}{12} mL^2 w_2$$

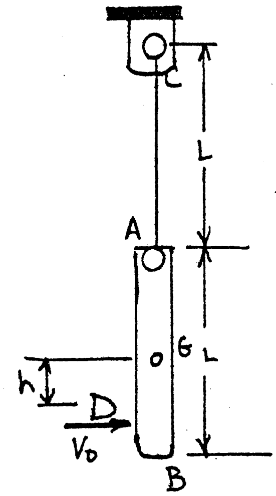
$$w_2 = \frac{8}{17} w_1$$

A bullet weighing 40g is fired with a horizontal velocity of 600m/s into the 7kg wooden rod AB of length 1m. The rod, which is initially at rest, is suspended by a cord of length 1m. Knowing that h = .2m, determine the velocity of each end of the rod immediately after the bullet becomes embedded.



System + Ext = System

Momemta$_1$ Impulse$_{1-2}$ Momenta$_2$



$\overset{+}{\curvearrowleft}$ Moments about G: $M_B V_o h = I w_2$ (1)

$\overset{+}{\rightarrow}$ x components: $M_B V_o = M V_2$ (2)

From equation (2): $V_2 = \dfrac{M_B}{M} V_o$ (3)

From equation (1): $w_2 = \dfrac{M_B V_o h}{I} = \dfrac{M_B V_o h}{ML^2/12} = \dfrac{12 M_B V_o h}{L^2}$ (4)

<u>DATA:</u>

$M_B = .040\text{kg}$

$V_o = 600\text{m/s}$ $h = .2\text{m}$

$M = 7\text{kg}$

$L = 1\text{m}$

From EQ. (3)

$V_2 = \dfrac{.04}{7}(600) = 3.43\text{m/s} \rightarrow$

From EQ. (4)

$w_2 = \dfrac{(12)(.04)(600)(.2)}{1^2} = 57.6\text{ra}$

<u>KINEMATICS</u>

$V_A = V_2 + V_{A/G} = \left[V_2 \rightarrow\right] + \left[\frac{L}{2} w_2 \leftarrow\right]$

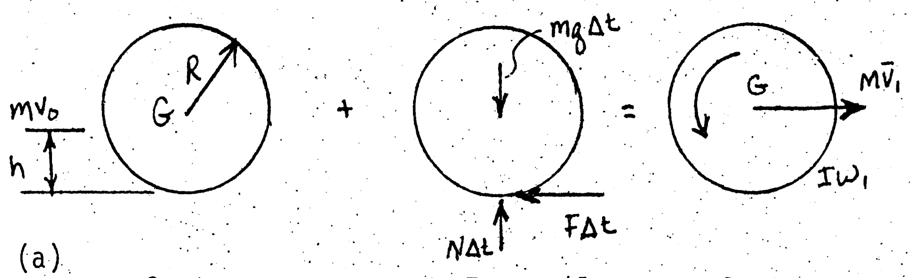
$= \left[3.43 \rightarrow\right] + \left[28.8 \leftarrow\right]$

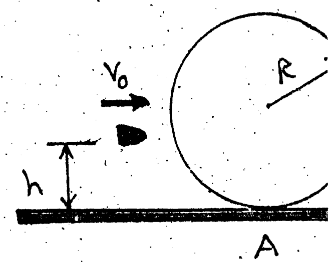
$V_A = 25.37 \leftarrow$

$V_B = V_2 + V_{B/G} = \left[V_2 \rightarrow\right] + \left[\frac{L}{2} w \rightarrow\right] = \left[3.43 \rightarrow\right] + \left[28.8 \rightarrow\right]$

$V_B = 32.23 \rightarrow$

A bullet of mass m is fired with a horizontal velocity v_o and at a height $h = R/2$ into a wooden disk of much larger mass M and radius R. The disk rests on a horizontal plane and the coefficient of friction between the disk and the plane is finite. (a) Determine the linear velocity $\bar{v}_1$ and the angular velocity $omega_1$ of the disk immediately after the bullet has penetrated the disk, (b) Describe the ensuing motion of the disk and determine its linear velocity after the motion has become uniform.





(a)

System + External = System

Momenta$_o$ Impulse Momenta$_1$

$+\uparrow$ y Components: $0 + N\Delta t - Mg\Delta t = 0$

$mg = N$

$\xrightarrow{+}$ x Components: $MV_o - F\Delta t = MV_1$

$MV_o - \mu Mg\Delta t = MV_1$

Since $\Delta t \approx 0$ $MV_o = MV_1$

(1) $V_1 = \dfrac{MV_o}{M}$

$+\circlearrowleft$ About G: $MV_o(R - h) - R(\mu Mg\Delta t) = Iw_1$

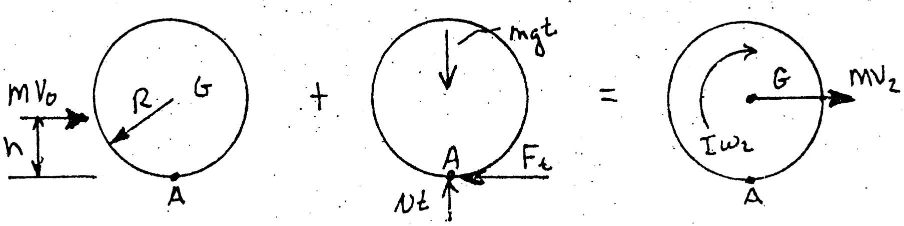
$\Delta t \approx 0$ $\quad MV_o(R - h) = \frac{1}{2} MR^2 w_1$

$w_1 = 2\,\dfrac{m}{M}\,\dfrac{R - h}{R^2}\,V_o$ (2)

$h = \dfrac{R}{2}$

$w_1 = \dfrac{MV_o}{MR}$

(b)



System + External = System

Momenta$_o$ Impulse Momenta$_2$

$w_2 = \dfrac{MV_o}{3MR}$

$V_2 = Rw_2$

$V_2 = \dfrac{MV_o}{3M} \rightarrow$

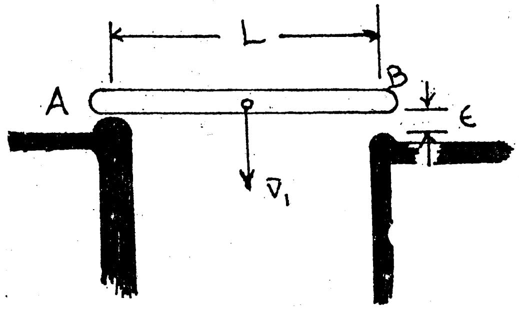
$+\circlearrowright$ Mom. About A:

$MV_o h + 0 = MV_2 R + Iw_2$

$MV_o(R/2) = (MRw_2)R + \frac{1}{2} MR^2 w_2$

$\frac{1}{2} MV_o = \frac{3}{2} MRw_2$

A uniform slender rod of length L is dropped onto rigid supports at A and B. Immediately before striking A the velocity of the rod is $\bar{v}_1$. Since support B is slightly lower than support A, the rod strikes A before it strikes B. Assuming perfectly elastic impact at both A and B, determine the angular velocity of the rod and the velocity of its mass center immediately after the rod (a) strikes support A, (b) strikes support B, (c) again strikes support A.



(a) $V_{A1} = V_1 \qquad e = 1$

$V_{A2} = -V_{A1} = -V_1$

$MV_1L/2 + 0 = Iw_2 + MV_2L/2$

$\frac{1}{2} MV_1L = \frac{1}{12} ML^2w_2 + \frac{1}{2} ML(\frac{1}{2} Lw_2 - V_1) \qquad I = \frac{1}{12} ML^2$

$V_1 = \frac{1}{3} Lw_2 \qquad V_2 = V_{A2} + V_{G/a2}$

$w_2 = \frac{3V_1}{L} \qquad = -V_1 + \frac{L}{2} w_2$

$V_G = \frac{L}{2} w_2 - V_1$

$V_G = \frac{1}{2} V_1$

(b) $V_{B2} = Lw - V_1 = 3V_1 - V_1 \qquad V_3 = V_{B3} + V_{G/B3} = -2V_1 - \frac{1}{2} Lw_3$

$V_{BL} = 2V_1 \qquad e = 1$

$V_{B3} = -2V_1$

$Iw_2 - (MV_2) \frac{L}{2} = Iw_3 + (MV_3) \frac{L}{2}$

$\frac{1}{12} ML^2(\frac{3V_1}{L}) - \frac{1}{4} V_1LM = \frac{1}{12} ML^2w_3 + m(2V_1 + \frac{L}{2} w_3) \frac{L}{2}$

$0 = \frac{1}{3} ML^2w_3 + MV_1L$

$w_3 = -\frac{3V_1}{L}$

c) $V_{A3} = V_{B3} + V_{A/B3} = -2V_1 + 3V_1$

$V_{A3} = V_1 \qquad e = 1$

$V_{A4} = -V_1$

$$V_4 = V_{A4} + V_{G/A4} = -V_1 + \frac{1}{2} Lw_4$$

$$Iw_3 + \frac{1}{2} LMV_3 = -Iw_4 + \frac{1}{2} MV_4 L$$

$$\frac{1}{12} ML^2\left(\frac{3V_1}{L}\right) + M\left(\frac{1}{2} MV_1\right) \frac{L}{2} = -\frac{1}{12} ML^2 w_4 + M\left(V_1 - \frac{L}{2}w_4\right) \frac{L}{2}$$

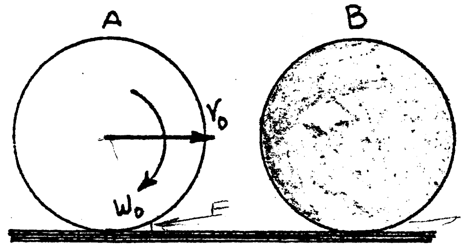
$$0 = -\frac{1}{3} ML^2 w_4$$

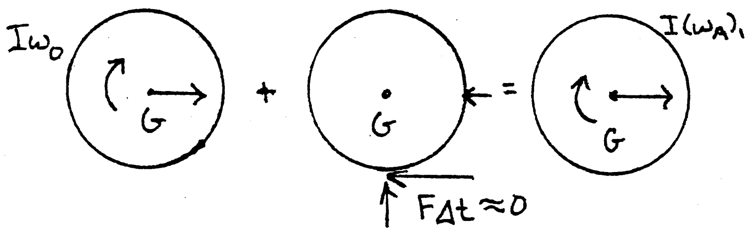
$$w_4 = 0$$

A sphere A of mass m and radius r rolls without slipping with a velocity v_0 on a horizontal plane. It hits squarely an identical sphere B which is at rest. Denoting by the coefficient of friction between the spheres and the plane, neglecting the friction between the spheres, and assuming perfectly elastic impact (e = 1), determine (a) the linear and angular velocity of each sphere immediately after impact, (b) the velocity of each sphere after it has started rolling uniformly, (c) discuss the special case when = 0.

(a) Immediately after Impact
(Friction forces are not impulsive)

CONSIDER SPHERE A



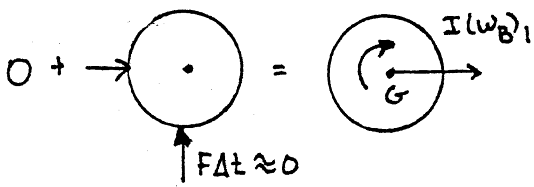


Sum of Moments About G:

$$Iw_0 = I(w_A)_1$$

$$(w_A)_1 = w_0 \circlearrowright = \frac{V_0}{r} \circlearrowright$$

CONSIDER SPHERE B

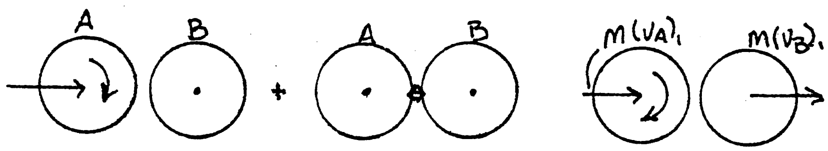


Sum Moments About G:

$$0 = I(w_B)_1$$

$$(w_B)_1 = 0$$

CONSIDER BOTH SPHERES



System Momenta$_0$ + External Impulse$_{0-1}$ = System Momenta$_1$

$+\rightarrow$ Components:

$$MV_0 = M(V_A)_1 + M(V_B)_1 \qquad (1)$$

Solve (1) and (2) Simultaneously:

$$(V_A)_1 = 0$$

$$(V_B)_1 = V_0 \rightarrow$$

Coefficient of Restitution (e = 1)

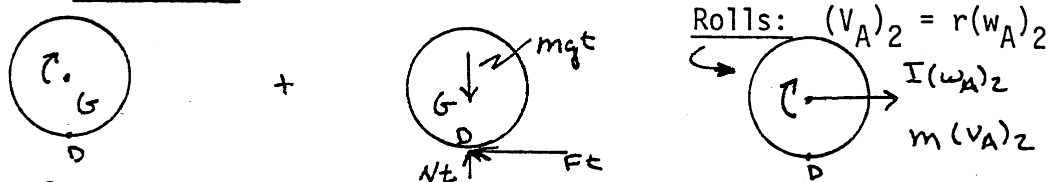
$$e = \frac{(V_B)_1 - (V_A)_1}{V_0 -} = 1 \qquad V_0 = (V_B)_1 - (V_A)_1 \qquad (2)$$

(b) Motion After Spheres Roll Uniformly.

Note: Here the time interval is not small and impulse of friction forces is included.

Sphere A:

Rolls: $(V_A)_2 = r(w_A)_2$



System Momenta$_1$ + System Ext. Impulse$_{1-2}$ = System Momenta$_2$

+ ↻ Moments about D: $Iw_o = I(w_A)_2 + M(V_A)_2 r$

$$\frac{2}{5}Mr^2 w_o{}^2 = \frac{2}{5}Mr^2(w_A)_2 + Mr(w_A)_2 r$$

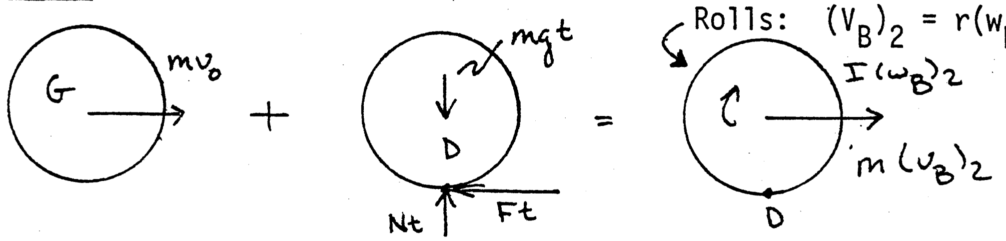
$$(w_A)_2 = \frac{2}{7}w_o \circlearrowright \blacktriangleleft$$

Since $rw_o = V_o$; we have: $(V_A)_2 = \frac{2}{7}w_o \rightarrow \blacktriangleleft$

$$(V_A)_2 = r(w_A)_2 = \frac{2}{7}rw_o$$

Sphere B

Rolls: $(V_B)_2 = r(w_B)_2$



System Moment a$_1$ + External Impulse$_{1-2}$ = System Monenta$_2$

+ ↻ Moments About D: $(MV_o)r = I(w_B)_2 + M(V_B)_2 r$

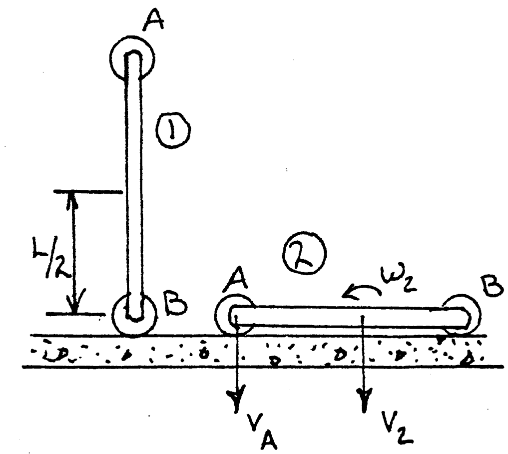
$$MV_o r = \frac{2}{5}Mr^2(w_B)_2 + Mr(w_B)_2 r$$

Recalling $\frac{V_o}{r} = w_o$ $\quad (w_B)_2 = \frac{5}{7}\frac{V_o}{r}$ $\quad (w_B)_2 = \frac{5}{7}w_o \circlearrowright \blacktriangleleft$

$(V_B)_2 = r(w_B)_2 = \frac{5}{7}rw_o$ $\qquad V_B = \frac{5}{7}V_o \rightarrow \blacktriangleleft$

(c) For $\mu = 0$ the motion of part (a) is the final motion

A slender rod of mass m and length l is held in the position shown. Roller B is given a slight push to the tight and moves along the horizontal plane, while roller A is constrained to move vertically. Determine the magnitudes of the impulses exerted on the rollers A and B as roller A strikes the ground. Assume perfectly plastic impact.



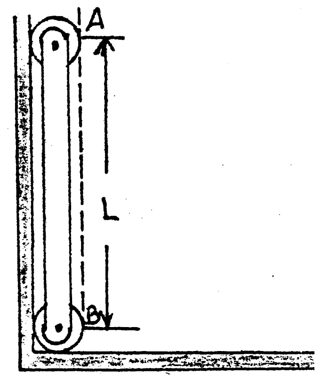
<u>Conservation of Energy</u>

Inst. center is @ B

$$V_2 = \frac{}{2} w_2$$

$$E_1 = 0$$

$${}_1W_2 = mg\,\frac{L}{2}$$



$$E_2 = \frac{1}{2} mV_2^{\,2} + \frac{1}{2} Iw_2^{\,2}$$

$$= \frac{1}{2} m\left(\frac{L}{2} w_2\right)^2 + \frac{1}{2}\left(\frac{1}{12} m\ell^2\right)w_2^{\,2}$$

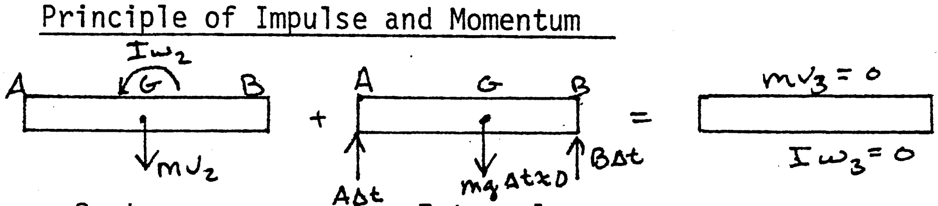
$$E_2 = \frac{1}{6} mL^2 w_2^{\,2} \qquad {}_1L_2 = 0$$

$$E_1 + {}_1W_2 = E_2 + {}_1L_2$$

$$mg\,\frac{L}{2} = \frac{1}{6} mL^2 w_2^{\,2}$$

$$w_2 = \sqrt{3g/\ell} \qquad (1)$$

<u>Principle of Impulse and Momentum</u>



System Momenta$_2$ + External Impulse$_{2-3}$ = System Momenta$_3$

+↰ Moments About B: $Iw_2 + mV_2\frac{L}{2} - (A\Delta t)L = 0$

But $I = \frac{1}{12} mL^2$ and $V_2 = \frac{1}{2}\ell w_2$, Thus

$$(A\Delta t)L = \frac{1}{12} m\ell^2 w_2 + m\left(\frac{L}{2}\right)^2 w_2 = \frac{1}{3} mL^2 w_2$$

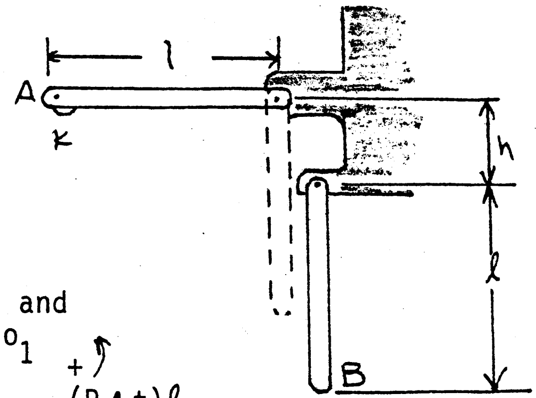
Substitute for w_2 from EQ (1)

$$A\Delta t = \frac{1}{3} mL\,\sqrt{3g/L} \qquad A\Delta t = m\sqrt{\frac{Lg}{3}}\ \uparrow$$

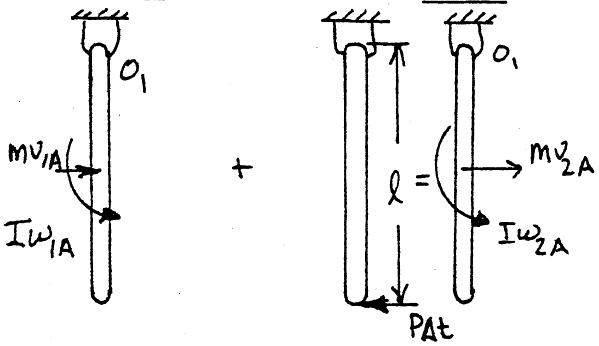
+↑ y compon.: $-mV_2 + A\Delta t + B\Delta t = 0$

$$-m\left[\frac{L}{2}\sqrt{\frac{3g}{L}}\right] + m\sqrt{\frac{Lg}{3}} + B\Delta t = 0 \qquad B\Delta t = m\sqrt{\frac{Lg}{12}}\ \uparrow$$

Two identical slender rods may swing freely from the pivots shown. Rod A is released from rest in a horizontal position and swings to a vertical position, at which time the small knob K strikes rod B which was at rest. If $h = 1/2\,\ell$ and $e = .5$, determine (a) the angle through which rod B will swing, (b) the angle through which rod A will rebound.



Impact   Rod A



Moment of imp. and momenta about O_1 $+\circlearrowleft$

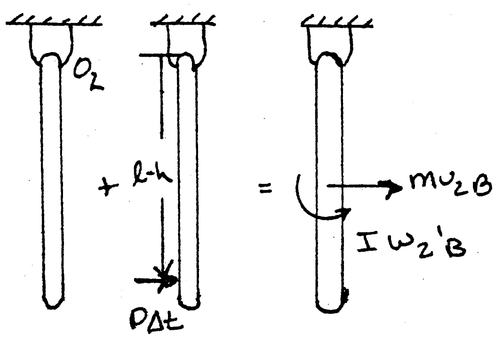
$$(mV_{1A})\frac{\ell}{2} + Iw_{1A} - (P\Delta t)\ell = (mV_{2A})\frac{\ell}{2} + Iw_{2A}$$

Since $V_A = \frac{\ell}{2}w_A$, $I = \frac{1}{12}m\ell^2$

$$\frac{1}{4}m\ell^2 w_{1A} + \frac{1}{12}m\ell^2 w_{1A} - (P\Delta t)\ell = \frac{1}{4}m\ell^2 w_2A + \frac{1}{12}m\ell^2 w_{2A}$$

$$\frac{1}{3}m\ell^2 w_{1A} - (P\Delta t)\ell = \frac{1}{3}m\ell^2 w_{2A} \qquad (1)$$

Rod B: Moments of imp. and momenta about O_2



$$+\circlearrowleft \quad P\Delta t(\ell - h) = (mV_{2B})\frac{\ell}{2} + Iw_{2B}$$

$$(P\Delta t)(\ell - h) = \frac{1}{3}m\ell^2 w_{2B} \qquad (2)$$

Eliminate P t from (1) and (2)

$$(\ell - h)(w_{1A} - w_{2A}) = \ell w_{2B} \qquad (3)$$

Relative Velocity of points of impact

$$(\ell - h)w_{2B} - \ell w_{2A} = e(\ell w_{1A}) \qquad (4)$$

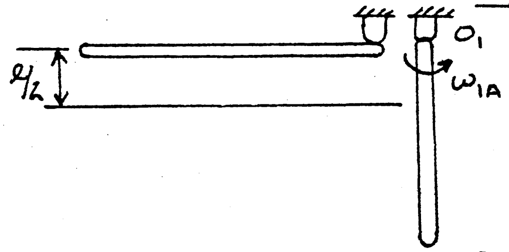
For $h = \frac{\ell}{2}$, EQ (3) yields: $\frac{1}{2}(w_{1A} - w_{2A}) = w_{2B}$ (5)

EQ (4) yields: $\frac{1}{2}w_{2B} - w_{2A} = e\,w_{1A}$ (6)

Substitute $e = .5$ and eliminate w_{2A} from (5) and (6)

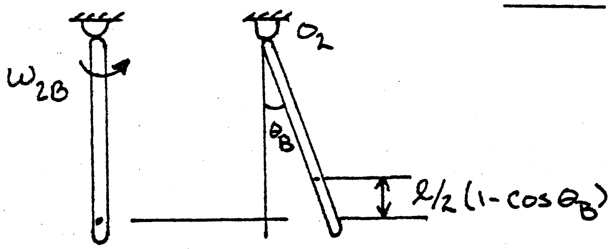
$$3w_{1A} = 5w_{2B} \qquad (7)$$

Conservation of Energy



Rod A: $E_1 + {}_1W_2 = E_2 + {}_1L_2$

$$mg\frac{\ell}{2} = \frac{1}{2} I_o (w_{1A})^2 \tag{8}$$



Rod B: $E_2 + {}_2W_3 = E_3 + {}_2L_3$

$$\frac{1}{2} I_o (w_{2B})^2 = mg\frac{\ell}{2}(1 - \cos\theta_B) \tag{9}$$

Substitute w_{1A}, from EQ (8) and (9), into EQ (7)

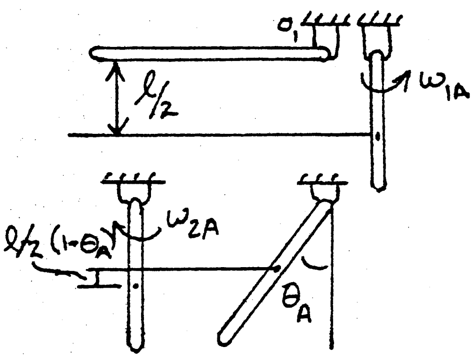
(a) $1 - \cos\theta_B = \left(\frac{w_{2B}}{w_{1A}}\right)^2 = \left(\frac{3}{5}\right)^2$ $\qquad \theta_B = 50.2^o$

(b) From EQ. (7) $w_{2B} = .6\, w_{1A}$; Subst. into EQ(6), with e = .5:

$$\frac{1}{2}(.6w_{1A}) - w_{2A} = \frac{1}{2} w_{1A}$$

$$w_{2A} = -\frac{1}{5} w_{1A} \tag{10}$$

Conservation of Energy: Rod A



$$E_1 + {}_1W_2 = E_2 + {}_1L_2$$

$$mg\,\ell/2 = \frac{1}{2} I_o (w_{1A})^2 \tag{11}$$

$$E_2 + {}_2W_3 = E_3 + {}_2L_3$$

$$\frac{1}{2} I_o (w_{2A})^2 = mg\,\ell/2(1 - \cos\theta_A) \tag{12}$$

Subst. w_{1A} from EQ (11) and W_{2A} from EQ (12) into into EQ (10)

$$(1 - \cos\theta_A) = \left(\frac{w_{2A}}{w_{1A}}\right) = (-.2)^2 = .004$$

$\mathrm{Cos}\,\theta_A = .96$ $\qquad \theta_A = 16.3^o$