

The motion of a cam is defined by the relation  $\theta = 2t^3 - t^2 + 4t - 7$  where  $\theta$  is expressed in radians and  $t$  is in seconds. Determine the angular coordinate the angular velocity and the angular acceleration of the cam when (a)  $t=0$ , (b)  $t= 3$  sec.

$$\theta = 2t^3 - t^2 + 4t - 7$$

$$\theta_0 = -7 \text{ radians}$$

$$\theta_3 = 50 \text{ radians}$$

$$w = \frac{d\theta}{dt} = 6t^2 - 2t + 4$$

$$w_0 = 4 \text{ rad/sec}$$

$$w_3 = 52 \text{ rad/sec}$$

$$\alpha = \frac{dw}{dt} = 12t - 2$$

$$\alpha_0 = -2 \text{ rad/sec}^2$$

$$\alpha_3 = 34 \text{ rad/sec}^2$$



The assembly shown rotates about the rod AC with a constant angular velocity of 5 rad/s. Knowing that at the instant considered, the velocity of corner D is downward, determine the velocity and acceleration of corner D.

$$\overline{AB} = (.24)\mathbf{i} + (.07)\mathbf{j}$$

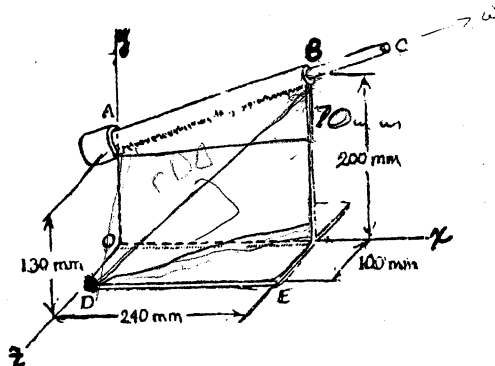
$$\overline{AB} = .25\text{m} \quad \alpha = 0$$

$$\overline{\omega}_{AB} = \frac{\overline{AB}}{AB} = (1/.25)(.24\mathbf{i} + .07\mathbf{j})$$

$$\omega = 5 \text{ rad/sec}$$

$$\overline{\omega} = \omega \overline{\omega}_{AB} = (4.8\mathbf{i} + 1.4\mathbf{j}) \text{ rad/sec}$$

$$\overline{r}_{D/A} = -.13\mathbf{j} + .1\mathbf{k}$$



$$\overline{v}_D = \overline{\omega} \times \overline{r}_{D/A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4.8 & 1.4 & 0 \\ 0 & -.13 & .1 \end{vmatrix} = (.14\mathbf{i} - .48\mathbf{j} - .624\mathbf{k}) \text{ m/s}$$

$$\overline{a}_D = \overline{\omega} \times \overline{v}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4.8 & 1.4 & 0 \\ .14 & -.48 & -.624 \end{vmatrix} = (-.874\mathbf{i} + 3.00\mathbf{j} - 2.5\mathbf{k}) \text{ m/s}^2$$

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DYNAMICS

PROBLEM N7/3

The rod ABCD has been bent as shown and may rotate about the line joining A and D. Knowing that the rod starts from rest in the position shown with a constant angular acceleration of  $14 \text{ rad/s}^2$  and that its initial acceleration of point B is upward, determine the initial acceleration of point C.

$$\overline{AD} = .6i + .2j - .3k$$

$$AD = .7$$

$$\overline{\text{Lamda}}_{AD} = (\overline{AD} / AD) = (6/7)i + (2/7)j - (3/7)k$$

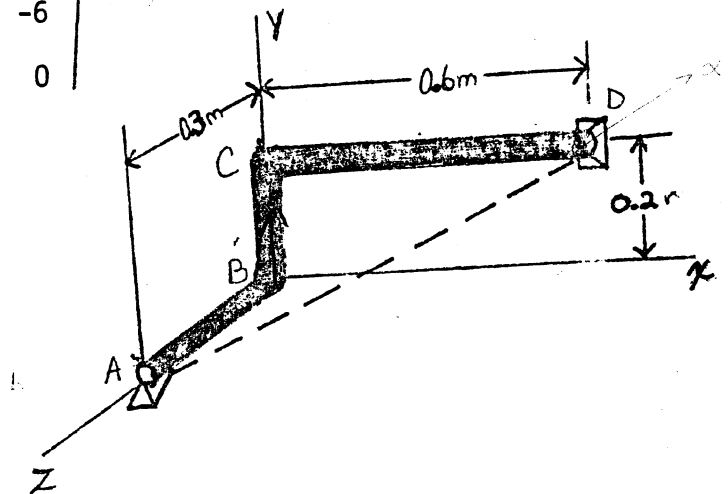
$$\alpha = 14 \text{ rad/s}^2 (\overline{\text{Lamda}}_{AD}) = 12i + 4j - .6k$$

$$\omega = 0$$

$$\overline{a}_c = \overline{\alpha} \times \overline{r}_{c/d} + \overline{\omega} \times (\overline{\omega} \times \overline{r}_{c/d})$$

$$\overline{a}_c = \overline{\alpha} \times \overline{r}_{c/d} = \begin{vmatrix} i & j & k \\ 12 & 4 & -6 \\ -.6 & 0 & 0 \end{vmatrix}$$

$$\overline{a}_c = (3.6j + 2.4k) \text{ m/s}^2$$



The friction wheel B executes 100 revolutions about its fixed shaft during the time interval  $t$ , while its angular velocity is being increased uniformly from 200 to 600 rpm. Knowing that wheel B rolls without slipping on the inside rim of wheel A, determine (a) the angular acceleration of wheel A; (b) the time interval  $t$ .

For Wheel B

$$\omega_0 = 200 \text{ rpm} = 200 \frac{2\pi}{60} \text{ rad/s}$$

$$\omega = 600 \text{ rpm} = 600 \frac{2\pi}{60} \text{ rad/s}$$

$$\theta_B = 100 \text{ rev} = 100(2\pi \text{ rad})$$

$$\omega^2 = \omega_0^2 + 2(\alpha_B)\theta_B$$

$$\left(600 \frac{2\pi}{60}\right)^2 = \left(200 \frac{2\pi}{60}\right)^2 + 2(\alpha_B)(100 \cdot 2\pi)$$

$$\alpha_B = (8/9)\pi = 2.79 \text{ rad/sec}^2$$

$$\omega = \omega_0 + (\alpha_B)t$$

$$\left(600 \frac{2\pi}{60}\right) = \left(200 \frac{2\pi}{60}\right) + 2.79 t$$

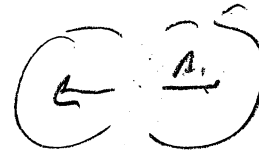
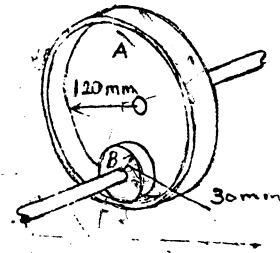
$$t = 15 \text{ sec.}$$

$$a_{A_t} = a_{B_t}$$

$$r_A(\alpha_A) = r_B(\alpha_B)$$

$$\alpha_A = (r_B/r_A)(\alpha_B) = \frac{30}{120} (2.79)$$

$$= 0.698 \text{ rad/sec}^2$$



$$a_t = \alpha \cdot r$$

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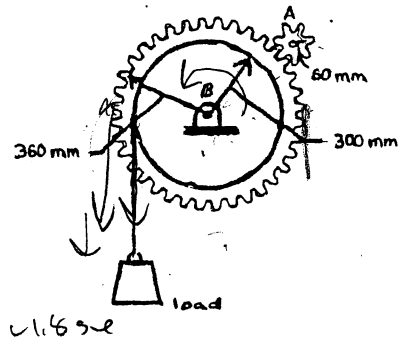
DYNAMICS

PROBLEM N7/5

The system shown starts from rest at  $t=0$  and accelerates uniformly. Knowing that at  $t=4s$  the velocity of the load is  $4.8 \text{ m/s}$  downward, determine (a) the angular acceleration of gear A, (b) the number of revolutions executed by gear A during the  $4 \text{ s}$  interval.

$$\bar{a}_{\text{load}} = (v/t) = \frac{4.8}{4} = 1.2 \text{ m/s}^2 \text{ downward}$$

$$(a_{\text{gear tooth}})_t = \frac{360}{300} a_{\text{load}} = 1.44 \text{ m/s}^2 \text{ downward}$$



$$(a_{\text{gear tooth}})_t = r (\alpha_A)$$

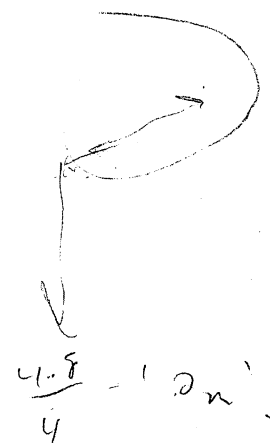
$$1.44 = 0.06(\alpha_A)$$

$$\alpha_A = 24 \text{ rad/sec clockwise}$$

$$\theta_A = (1/2)(\alpha_A)t^2 = (1/2)(24)4^2 = 192 \text{ rad}$$

$$(\alpha_T) 360 = 300 (\alpha_L)$$

$$\theta_A = 30.6 \text{ rev.}$$



The two friction wheels A and B are to be brought together. Wheel A has an initial angular velocity of 600 rpm clockwise and will coast to rest in 40 s, while wheel B is initially at rest and is given a constant counterclockwise angular acceleration of 2 rad/sec/sec. Determine (a) at what time the wheels may be brought together if they are not to slip, (b) the angular velocity of each wheel as contact is made.

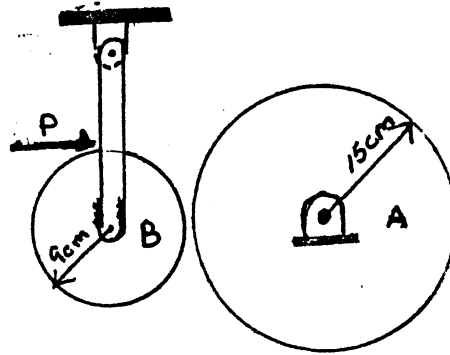
Wheel A

$$w_0 = 600 \text{ rpm} = 62.83 \text{ rad/s}$$

$$w = w_0 + (\alpha)t$$

$$0 = 62.83 + (\alpha_A)40$$

$$\alpha_A = 1.571 \text{ rad/s}^2$$



Thus clockwise is considered positive  $w_A = 62.83 - 1.571 t$

Wheel B

$$w_0 = 0, (\alpha_B) = 2 \text{ rad/s}^2 \quad \text{If ccw is positive} \quad w_B = 2t$$

$$(a) \quad r_A w_A = r_B w_B$$

$$(15)(62.83 - 1.571t) = 9(2t)$$

$$942.45 - 23.565t = 18t$$

$$t = 22.7 \text{ sec.}$$

$$(b) \quad w_A = 62.83 - 1.571(22.7)$$

$$w_B = 2t = 2(22.7)$$

$$w_A = 27.2 \text{ rad/sec cw}$$

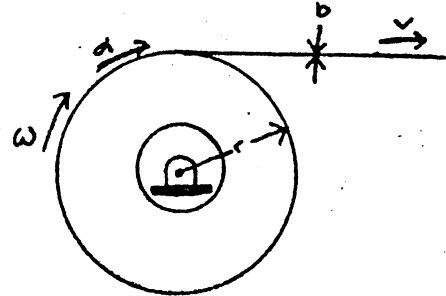
$$w_B = 45.2 \text{ rad/sec ccw}$$

In a continuous printing process, paper is drawn into the presses at a constant speed  $v$ . Denoting by  $r$  the radius of paper on the roll at any given time and  $b$  the thickness of the paper, derive the expression for the angular acceleration of the paper roll.

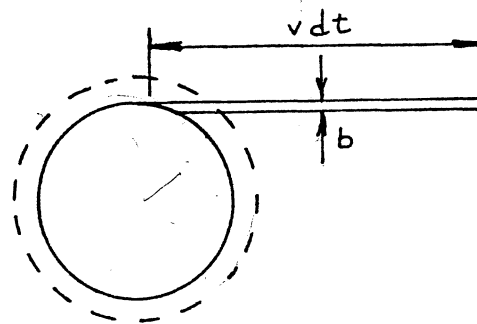
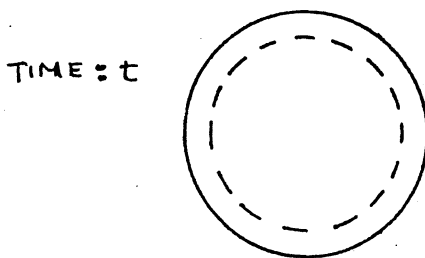
We have  $\omega = (v/r)$

Since  $v = \text{constant}$

$$\begin{aligned} \alpha &= \frac{d\omega}{dt} = \frac{d}{dt} \left( \frac{v}{r} \right) = v \left( \frac{d}{dr} \right) \left( \frac{1}{r} \right) \frac{dr}{dt} \\ &= \frac{-v}{r^2} \frac{dr}{dt} \quad (1) \end{aligned}$$



We note the amount of paper unrilled in time  $dt$  (assuming unit width) is:



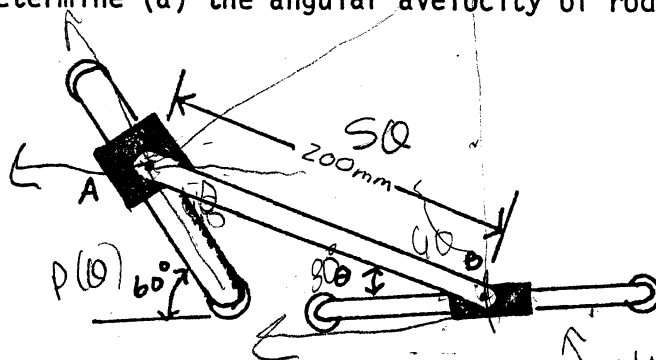
$$\text{Amount} = 2\pi r(-dr) = b(v dt) \quad (2)$$

Substituting for  $dr/dt$  from (2) into (1)

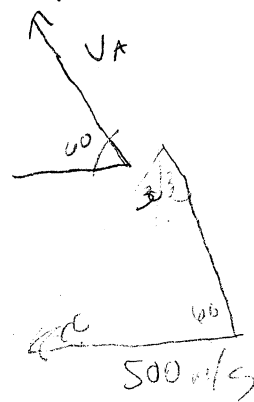
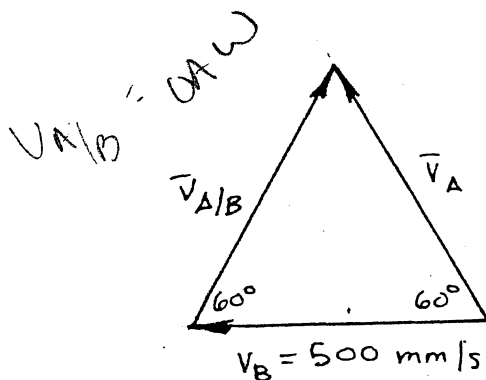
$$\begin{aligned} \alpha &= \frac{-v}{r^2} \left( \frac{-bv}{2\pi r} \right) \\ &= \frac{bv^2}{2\pi r^3} \end{aligned}$$

Collar B moves with a constant velocity of 500 mm/s to the left. At the instant when  $\theta = 30^\circ$ , determine (a) the angular velocity of rod AB, (b) the velocity of collar A.

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$



$$\vec{v}_A \angle 60^\circ = 500 \rightarrow + 200 \angle 60^\circ$$



Equilateral Triangle

$\omega$

(a)  $v_{A/B} = 200\omega = 500 \text{ mm/s}$

$\omega = 2.5 \text{ rad/s}$  clockwise

(b)  $v_A = 500 \text{ mm/s}$

$v_B = \omega \cdot r$



DYNAMICS

PROBLEM N7/9

The plate shown moves in the xy plane. Knowing that  $v_{ax} = 80 \text{ mm/s}$  and  $v_{by} = 200 \text{ mm/s}$  and  $v_{cy} = -40 \text{ mm/s}$ , determine:

- (a) the angular velocity of the plate
- (b) the velocity of point A.

$$\bar{r}_{C/B} = 120 \text{ i} + 240 \text{ j}$$

$$\bar{v}_C = \bar{v}_B + \bar{v}_{C/B}$$

(a)  $v_{Cx} - 40j = (v_{Bx} \text{ i} + 200j) + \bar{w} \times \bar{r}_{C/B}$

$$v_{Cx} - 40j = (v_{Bx} \text{ i} + 200j) + w \text{ k} \times (120 \text{ i} + 240 \text{ j})$$

$$v_{Cx} - 40j = v_{Bx} \text{ i} + 200j + 120w \text{ j} - 240w \text{ i}$$

j:  $-40 = 200 + 120w$

$$w = -2 \text{ rad/sec} = 2 \text{ rad/sec clockwise}$$

(b)  $\bar{r}_{A/B} = (-120) \text{ i} + 120 \text{ j}$

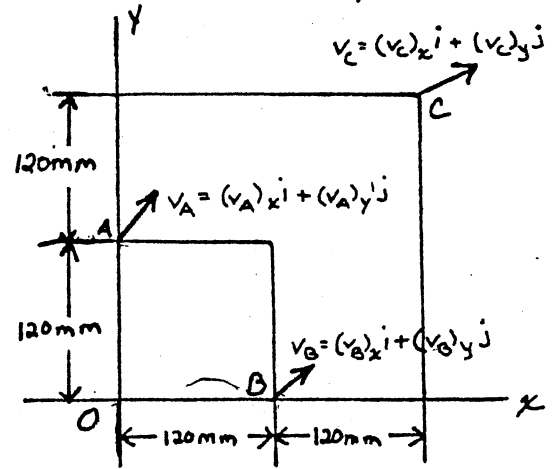
$$\bar{v}_A = \bar{v}_{A/B} + \bar{v}_B = \bar{v}_B + \bar{w} \times \bar{r}_{A/B}$$

$$80 \text{ i} + v_{Ay} \text{ j} = v_{Bx} \text{ i} + 200 \text{ j} - 2 \text{ k} \times (-120 \text{ i} + 120 \text{ j})$$

$$80 \text{ i} + v_{Ay} \text{ j} = v_{Bx} \text{ i} + 200 \text{ j} + 240 \text{ j} + 240 \text{ i}$$

j:  $v_{Ay} = 200 + 240 = 440 \text{ mm/s}$

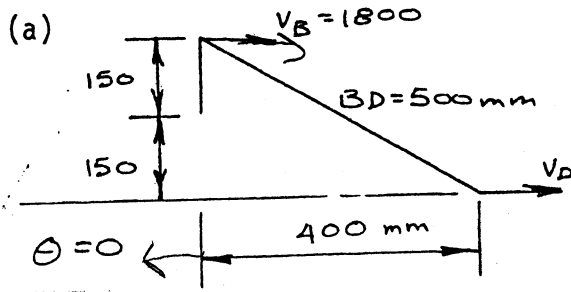
$$\bar{v}_A = 80 \text{ i} + 440 \text{ j} \text{ (mm/s)}$$



Crank AB has a constant angular velocity of 12 rad/s clockwise. Determine the angular velocity of rod BD and the velocity of collar D when (a)  $\theta = 0^\circ$ , (b)  $\theta = 90^\circ$ , (c)  $\theta = 180^\circ$ .

$$\bar{\omega}_{AB} = 12 \text{ rad/s} \curvearrowright$$

$$\bar{v}_B = r \omega_{AB} = (150)(12) = 1800 \text{ mm/s}$$



$$\bar{v}_D = \bar{v}_B + \bar{v}_{D/B}$$

$$\left[ \bar{v}_D \rightarrow \right] = \left[ 1800 \rightarrow \right] + \left[ 500 \omega_{BD} \right]$$

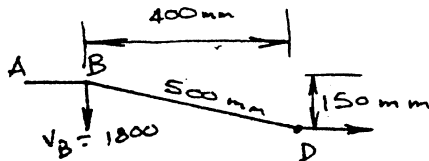
Thus:  $+ \uparrow 500 \omega_{BD} = 0$

$$\omega_{BD} = 0$$

$$\rightarrow v_D = 1800 \text{ mm/s}$$

$$v_D = 1800 \text{ mm/s} \rightarrow$$

(b)  $\theta = 90^\circ$



$$\bar{v}_D = \bar{v}_B + \bar{v}_{D/B}$$

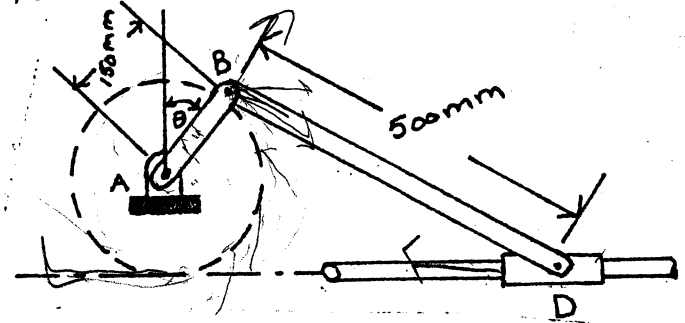
$$\bar{v}_D \rightarrow = \left[ 1800 \downarrow \right] + \left[ 500 \omega_{BD} \right]$$

$$v_D = (1800) \left( \frac{150}{400} \right) = 675$$

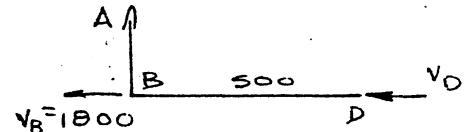
$$500 \omega_{BD} = 1800 \left( \frac{500}{400} \right)$$

$$\omega_{BD} = 4.5 \text{ rad/s} \curvearrowright$$

$$v_D = 675 \text{ mm/s} \rightarrow$$



(c)  $\theta = 180^\circ$



$$\bar{v}_D = \bar{v}_B + \bar{v}_{D/B}$$

$$\left[ \bar{v}_D \leftarrow \right] = \left[ 1800 \leftarrow \right] + \left[ 25 \text{ BD} \rightarrow \right]$$

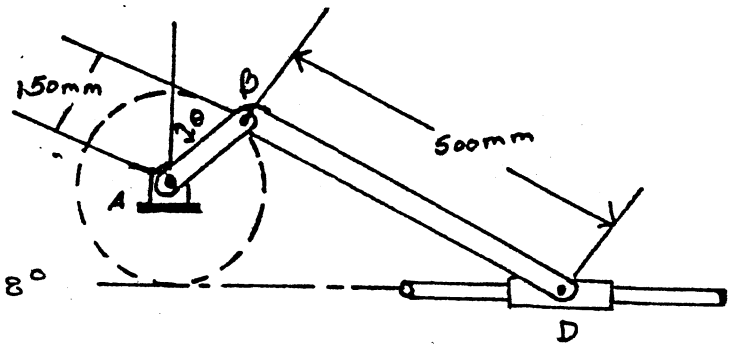
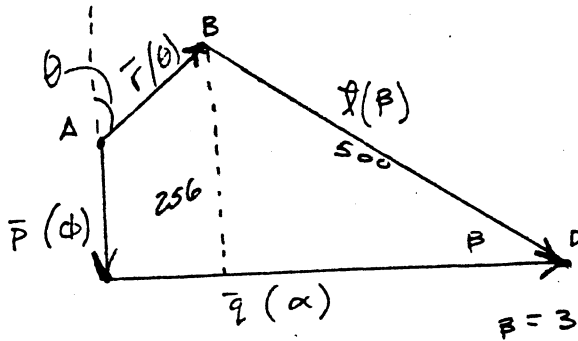
$$\uparrow + 25 \text{ BD} = 0$$

$$\text{BD} = 0$$

$$\leftarrow + v_D = 1800$$

$$\bar{v}_D = 1800 \text{ mm/s} \leftarrow$$

Crank AB has a constant angular velocity of 12 rad/s clockwise. Determine the angular velocity of rod BD and the velocity of collar D when  $\theta = 45^\circ$ .



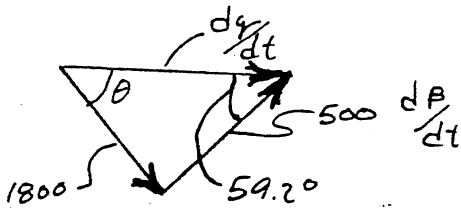
$$\bar{r} + \bar{l} = \bar{p} + \bar{q}$$

$$r\bar{u}_r + l\bar{u}_l = p\bar{u}_p + q\bar{u}_q$$

$$\frac{dr}{dt}\bar{u}_r + r\frac{d\theta}{dt}\bar{u}_\theta + \frac{dl}{dt}\bar{u}_l + l\frac{d\beta}{dt}\bar{u}_\beta = \frac{dp}{dt}\bar{u}_p + p\frac{d\phi}{dt}\bar{u}_\phi + \frac{dq}{dt}\bar{u}_q + q\frac{d\alpha}{dt}\bar{u}_\alpha$$

$$r\frac{d\theta}{dt}\bar{u}_\theta + l\frac{d\beta}{dt}\bar{u}_\beta = \frac{dq}{dt}\bar{u}_q$$

$$150(12) + 500\frac{d\beta}{dt} = \frac{dq}{dt}$$



$$\frac{\sin 59.2}{1800} = \frac{\sin 45}{500(d\beta/dt)}$$

$$\frac{\sin 45}{500(2.96)} = \frac{\sin (180-(45+59.2))}{dq/dt}$$

$$\frac{d(\beta)}{dt} = \omega_{BD} = 2.96 \text{ rad/s}$$

$$\frac{dq}{dt} = 2029.1 \text{ mm/s}$$

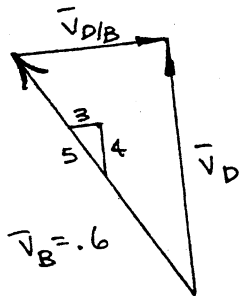
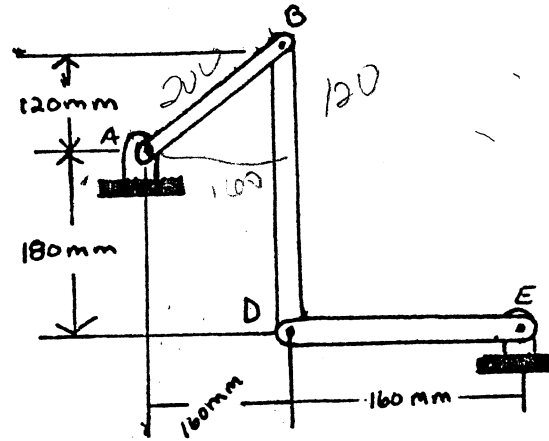
In the position shown, bar AB has a constant angular velocity of 3 rad/s counterclockwise. Determine the angular velocity of bars BD and DE.

Bar AB:  $\bar{v}_B = (.2\text{m})(3 \text{ rad/s}) = .6 \text{ m/s}$

BAR DE:  $\bar{v}_D = .16\text{m} (\omega_{DE}) = .16\omega_{DE}$

$$\bar{v}_D = \bar{v}_B + \bar{v}_{D/B}$$

$$\bar{v}_D = .6 + \bar{v}_{D/B}$$



$$\frac{.6}{5} = \frac{\bar{v}_D}{4} = \frac{\bar{v}_{D/B}}{3}$$

$$v_D = .48 \text{ m/s} = .16\omega_{DE}$$

$$\omega_{DE} = 3 \text{ rad/s clockwise}$$

$$v_{D/B} = .36 \text{ m/s} = .3 \omega_{BD}$$

$$\omega_{BD} = 1.2 \text{ rad/s counterclockwise}$$

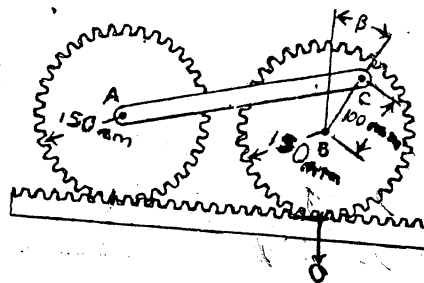
## DYNAMICS

## PROBLEM N7/13

Two gears, each of 300 mm diameter, are connected by a 450 mm rod AC. Knowing that the center of gear B has a constant velocity of 1 m/s to the right, determine the velocity of the center of gear A and the angular velocity of the connecting rod if  $\beta = 0$ .

$$\omega_B = v_B/r = (1000)/150 = 6.67 \text{ rad/s cw}$$

(a)  $\beta = 0$



Gear B:

$$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$$

$$\vec{v}_C = 1000\mathbf{i} + (100)(6.67)\mathbf{i}$$

$$\vec{v}_C = 1667 \text{ mm/s } \mathbf{i}$$

Rod AC

$$\vec{v}_C = \vec{v}_A + \vec{v}_{C/A}$$

$$1667 \mathbf{i} = \vec{v}_A \mathbf{i} + 450 \omega_{AC} \text{ (perpendicular to AC)}$$

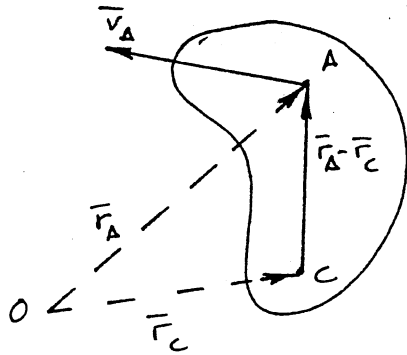
$$\omega_{AC} = 0$$

$$\vec{v}_A = 1667 \text{ mm/s}$$

Denoting by  $\bar{r}_A$  the position vector of a point A of a rigid slab which moves in plane motion, show that the position vector  $\bar{r}_C$  of the instantaneous center of rotation is

$$\bar{r}_C = \bar{r}_A + (\bar{\omega} \times \bar{v}_A)/\omega^2$$

where  $\omega$  is the angular velocity of the slab and  $v_A$  the velocity of point A.



Let C be the instantaneous center of of the slab. The velocity of A may be written as:

$$\begin{aligned}\bar{v}_A &= \bar{v}_C + \bar{v}_{A/C} \\ &= \bar{v}_C + \bar{\omega} \times (\bar{r}_A - \bar{r}_C)\end{aligned}$$

or, since  $v_C = 0$

$$\bar{v}_A = \bar{\omega} \times (\bar{r}_A - \bar{r}_C)$$

Cross multiply each member by  $\bar{\omega}$ :

$$\bar{\omega} \times \bar{v}_A = \bar{\omega} \times (\bar{\omega} \times (\bar{r}_A - \bar{r}_C))$$

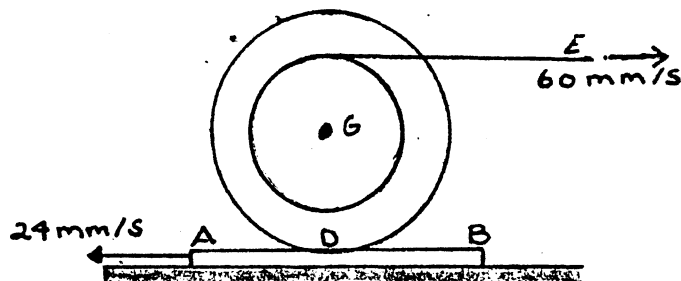
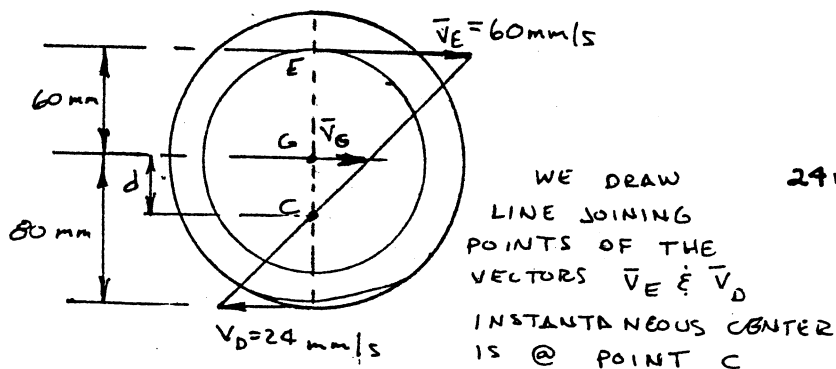
Since  $\bar{\omega}$  is perpendicular to the plane containing  $\bar{r}_A - \bar{r}_C$ , cross-multiplying  $\bar{r}_A - \bar{r}_C$  twice by  $\bar{\omega}$  is equivalent to multiplying  $\bar{r}_A - \bar{r}_C$  by  $\omega^2$  and rotating it through  $180^\circ$ :

$$\bar{\omega} \times \bar{v}_A = -\omega^2(\bar{r}_A - \bar{r}_C)$$

Solving for  $\bar{r}_C$

$$\bar{r}_C = \bar{r}_A + (\bar{\omega} \times \bar{v}_A)/\omega^2$$

A double pulley rolls without sliding on the plate AB which moves to the left at a constant speed of 24 mm/s. The 60 mm radius inner pulley is rigidly attached to the 80 mm radius outer pulley. Knowing that cord E is pulled at a constant speed of 60 mm/s as shown, determine (a) the angular velocity of the pulley, (b) the velocity of the center G of the pulley.



Similar triangles

$$\frac{v_E}{60+d} = \frac{v_D}{80-d} ; \quad \frac{60}{60+d} = \frac{24}{80-d}$$

$$d = 40 \text{ mm}$$

(a) angular velocity:

$$v_E = (CE)\omega ; \quad 60 \text{ mm/s} = (60+40)\omega$$

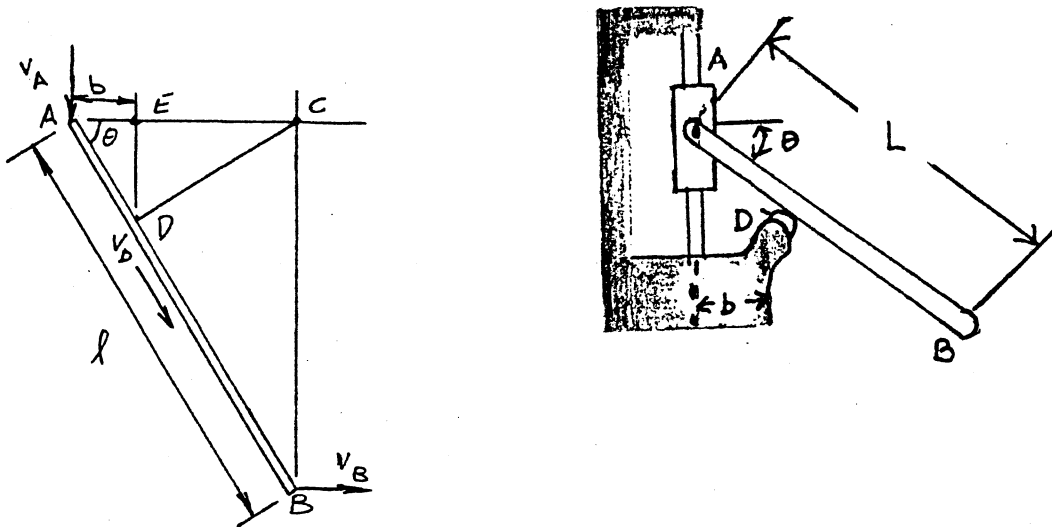
$$60 = 100\omega$$

$$\bar{\omega} = .6 \text{ rad/s } \text{ cw}$$

(b) Velocity of G

$$v_G = (CG)\omega = 40(.6) = 24 \text{ mm/s in the plus x direction}$$

Collar A slides downward with a constant velocity  $v_A$ . Determine the angle  $\theta$  corresponding to the position of rod AB for which the velocity of B is horizontal.



For  $v_B$  to be horizontal, the instant center C must be located directly above B. Since C must also be on perpendicular lines from  $v_A$  and  $v_D$ ; we must have the geometry shown

In triangle ABC:  $AC = L \cos \theta$

In triangle ACD  $AD = AC \cos \theta$

$$AD = L \cos^2 \theta$$

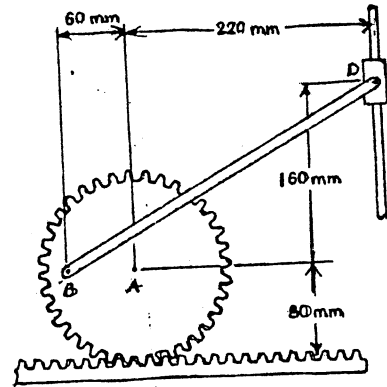
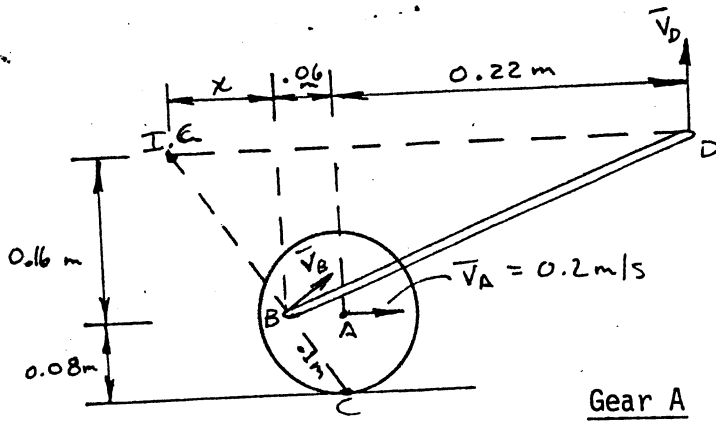
In triangle AED  $b = AD \cos \theta = L \cos^3 \theta$

or:

$$\cos^3 \theta = b/L$$



At the instant shown, the velocity of the center of the gear is 200 mm/s to the right. Determine (a) the velocity of B, (b) the velocity of collar D.



Instant center located at C.

$$w_A = v_A / (AC) = .2 / 0.08 = 2.5 \text{ rad/s cw}$$

(a) Velocity of B

$$v_B = (BC)w = .1(2.5) = .25 \text{ m/s } 36.9^\circ$$

(b) Velocity of D (Rod BD)

Instant center shown as IC in diagram at left

$$\frac{B(I.C.)}{BC} = \frac{x}{BA} = \frac{.16}{.08}$$

$$B(I.C.) = .2 \text{ m}$$

$$x = .12 \text{ m}$$

$$D(I.C.) = .12 + .06 + .22 = .40$$

$$v_B = B(I.C.)w_{BD}j$$

$$.25 = .20w_{BD}$$

$$w_{BD} = 1.25 \text{ rad/s ccw}$$

$$v_D = D(I.C.)w_{BD}$$

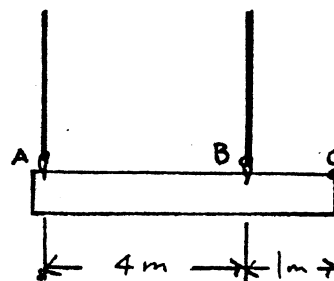
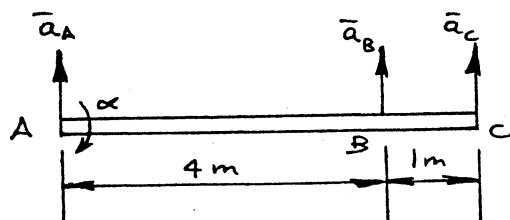
$$= .40(1.25)$$

$$= .5 \text{ m/s upward}$$

## DYNAMICS

## PROBLEM N7/18

A 5 m steel beam is lowered by means of two cables unwinding at the same speed from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow down the unwinding motion. At the instant shown, the deceleration of the cable attached at A is  $5 \text{ m/s}^2$  while that of the cable attached at B is  $3 \text{ m/s}^2$ . Determine (a) the angular acceleration of the beam, (b) the acceleration of point C.



$$(a) \bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$$

$$3\mathbf{j} = 5\mathbf{j} + -4(\alpha)\mathbf{j}$$

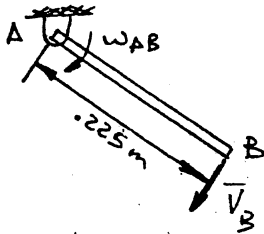
$$\alpha = .5 \text{ rad/s}^2 \quad \text{cw}$$

$$(b) \bar{a}_C = \bar{a}_A + \bar{a}_{C/A}$$

$$= 5\mathbf{j} + 5(.5)(-\mathbf{j}) = 5\mathbf{j} - 2.5\mathbf{j}$$

$$= 2.5 \mathbf{j}$$

Arm AB rotates with a constant angular velocity of 120 rpm clockwise. Knowing that gear A goes not rotate, determine the acceleration of the tooth of gear B which is in contact with gear A.

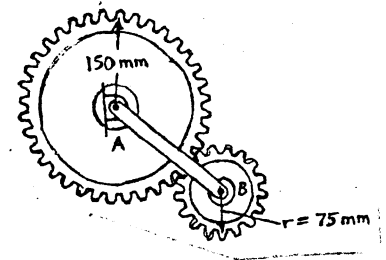
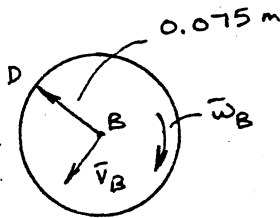
ROD AB

Angular velocities are constant.

$$\omega_{AB} = 120 \text{ rpm} = 12.57 \text{ rad/s}$$

$$\bar{v}_B = r \omega_{AB} = (.225) \omega_{AB}$$

$$\bar{a}_B = r \omega_{AB}^2 = (.225) \omega_{AB}^2$$

GEAR AB

Since Gear B Rolls on the Fixed Gear A

$$v_D = 0 \quad (D \text{ is inst. center})$$

$$\bar{\omega}_B = \frac{v_B}{BD} = \frac{.225 \omega_{AB}}{.075} = 3 \omega_{AB}$$

$$\bar{a}_D = \bar{a}_B + \bar{a}_{D/B}$$

$$\bar{a}_D = \left[ .225 \omega_{AB}^2 \right] + \left[ 0.075 \omega_B^2 \right]$$

$$= \left[ .225 \omega_{AB}^2 \right] + \left[ 0.075 (3 \omega_{AB})^2 \right]$$

$$= .450 \omega_{AB}^2$$

$$= .450 (12.57)^2$$

$$\bar{a}_D = 71.1 \text{ m/s}^2$$

Show that the acceleration of the instantaneous center of rotation of the slab is zero if and only if,

$$\bar{a}_A = (\alpha/w)\bar{v}_A + \bar{\omega} \times \bar{v}_A$$

Where  $\alpha = \alpha(k)$  is the angular acceleration of the slab.

Since we want  $\bar{a}_C = 0$ , we write

$$\bar{a}_C = \bar{a}_A + \bar{a}_{C/A} = 0 \quad (1)$$

$$\bar{a}_{C/A} = \alpha \times \bar{r}_{C/A} - w^2 \bar{r}_{C/A} \quad (2)$$

From problem r/14

$$\bar{r}_{C/A} = \bar{r}_C - \bar{r}_A = (\bar{\omega} \times \bar{v}_A) / w^2$$

Equation (2) becomes

$$\bar{a}_{C/A} = \alpha \times (\bar{\omega} \times \bar{v}_A) / w^2 - \bar{\omega} \times \bar{v}_A$$

But  $\alpha = \alpha(k)$  and  $\omega = \omega k$  ( $\bar{\omega} = \omega k$ )

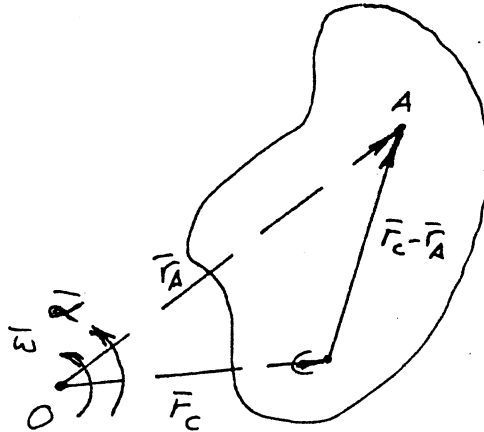
$$\bar{a}_{C/A} = \frac{\alpha}{w} (k \times (k \times \bar{v}_A)) - \bar{\omega} \times \bar{v}_A$$

Since  $k$  is perpendicular to  $\bar{v}_A$

$$\bar{a}_{C/A} = -\frac{\alpha}{w} \bar{v}_A - \bar{\omega} \times \bar{v}_A$$

Substituting into (1) and solving for  $\bar{a}_A$

$$\bar{a}_A = \frac{-\alpha}{w} \bar{v}_A + \bar{\omega} \times \bar{v}_A$$



Derive an expression for the angular acceleration of the rod AB in terms of  $v_B$ ,  $\theta$ ,  $L$  and  $\beta$ , knowing that the acceleration of point B is zero.

Law of Sines

$$\frac{x_B}{\sin \theta} = \frac{L}{\sin(\beta)}$$

$$x_B = \frac{L}{\sin(\beta)} \sin \theta$$

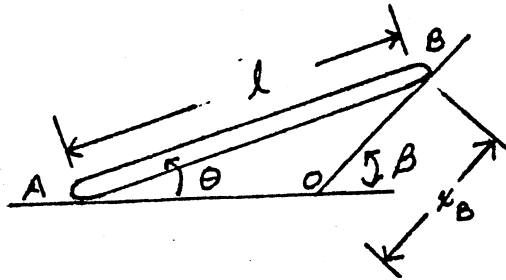
$$v_B = \frac{dx_B}{dt} = (L/\sin(\beta)) \cos \theta \frac{d\theta}{dt} = (L/\sin(\beta)) \cos \theta \omega$$

$$\omega = \frac{v_B \sin(\beta)}{L \cos \theta}$$

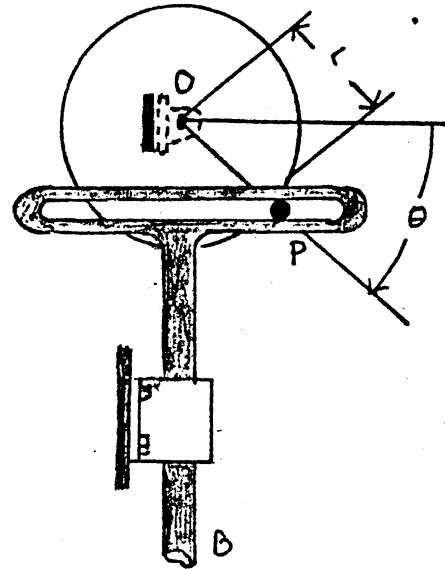
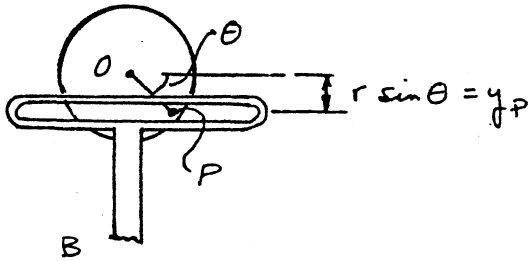
$$\alpha = \frac{d\omega}{dt} = \frac{v_B \sin(\beta)}{L} \frac{\sin \theta}{\cos^2 \theta} \frac{d\theta}{dt}$$

$$= \frac{v_B \sin(\beta) \sin \theta}{L \cos^2 \theta} \frac{v_B \sin(\beta)}{L \cos \theta}$$

$$= \left( \frac{v_B \sin(\beta)}{L} \right) \frac{\sin \theta}{\cos^3 \theta}$$



The drive disk of the Scotch crosshead mechanism shown has an angular velocity  $\omega$  and an angular acceleration  $\alpha$ , both directed clockwise. Derive an expression (a) for the velocity of point B, (b) the acceleration of point B.



$$(a) \ y_B = y_P + \text{constant}$$

$$= r \sin \theta + \text{constant}$$

$$v_B = dy_B/dt = r \cos \theta \frac{d\theta}{dt}$$

$$= r \omega \cos \theta$$

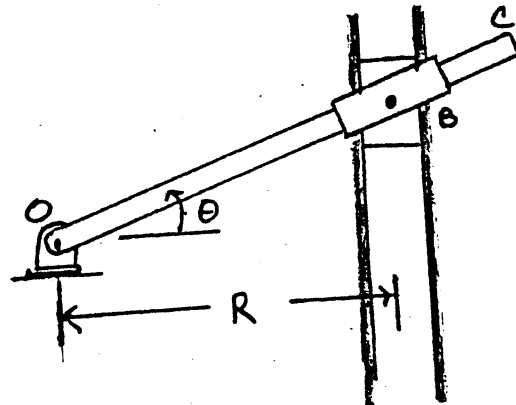
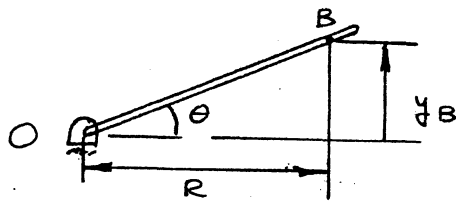
$$(b) \ a_B = dv_B/dt = r \frac{d\omega}{dt} \cos \theta - r \omega \sin \theta \frac{d\theta}{dt}$$

$$= r (\alpha) \cos \theta - r \omega^2 \sin \theta$$

## DYNAMICS

## PROBLEM N7/23

Collar B slides along rod OC and is attached to a sliding block which moves in a vertical slot. Knowing that rod OC rotates with an angular velocity  $\omega$  and with angular acceleration  $\alpha$ , both counterclockwise, derive an expression for the velocity and acceleration of collar B.



$$y_B = R \tan \theta$$

$$v_B = dy_B/dt = R \sec^2 \theta \frac{d\theta}{dt}$$

$$\approx R \omega \sec^2 \theta$$

$$a_B = dv_B/dt = d/dt(R\omega \sec^2 \theta)$$

$$\approx R \frac{d\omega}{dt} \sec^2 \theta + 2R \omega \sec^2 \theta \tan \theta \frac{d\theta}{dt}$$

$$= R(\alpha) \sec^2 \theta + 2R \omega^2 \sec^2 \theta \tan \theta$$

$$= R \sec^2 \theta (\alpha + 2\omega^2 \tan \theta)$$