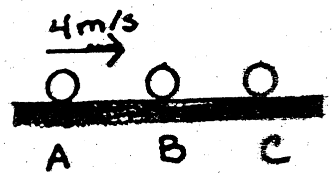
Two identical balls B and C are at rest when ball B is struck by a ball A of the same mass, moving with a velocity of 4 m/s. This causes a series of collisions between the various balls. Knowing e = .40, determine the velocity of each ball after all collisions have taken place.



1st Collision

A & B $v_A = 4m/s$

$\cancel{m_A} v_A + \cancel{m_B} v_B = \cancel{m_A} v_A' + \cancel{m_B} v_B'$

$v_A + \cancel{v_B}^{0} = v_A' + v_B'$

$$e = \frac{v_B' - v_A'}{v_A - v_B}$$

$.4v_A = v_B' - v_A'$

$v_B' = v_A' + 1.6$ eq (1)

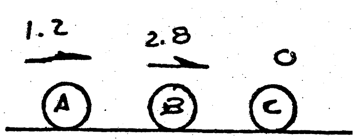
$4 = v_A' + v_B'$ eq (2)

solving eq. (1) and (2) simultaneously

$4 = v_A' + v_A' + 1.6$

$v_A' = 1.2$ m/s

$v_B' = 2.8$ m/s



2nd Collision

B & C

$v_B' + v_C' = v_B'' + v_C''$

$2.8 = v_B'' + v_C''$ eq. (3)

$.4\, v_B' = v_C'' - v_B''$

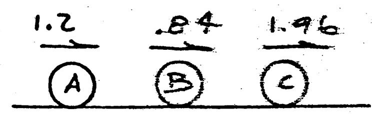
$v_C'' = v_B'' + 1.12$ eq. (4)

solving eq. (3) and (4) simultaneously

$2.8 = v_B'' + v_B'' + 1.12$

$v_B'' = .84$

$v_C'' = 1.96$



3rd Collision

A & B

$v_A'' + v_B'' = v_A''' + v_B'''$

$1.2 + (.84) = v_A''' + v_B'''$

$2.04 = v_A''' + v_B'''$ eq. (5)

$$.4 = \frac{v_B''' - v_A'''}{v_A'' - v_B''}$$

$.4(1.2-.84) = v_B''' - v_A'''$

$v_B''' = v_A''' + .144$ eq. (6)

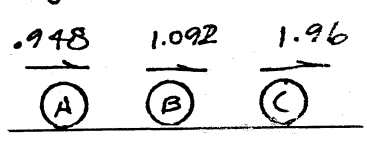
$2.04 = v_A''' + v_A''' + .144$

solving (5) and (6)

$v_A''' = .948$ m/s

$v_B''' = 1.092$ m/s

$v_C''' = 1.96$ m/s



A system consists of three particles A, B, and C. If m_A = 1 kg, m_B = 2 kg and m_C = 3 kg and the velocities of the particles are expressed in meters per second as $v_A = 3i - 2j + 4k$, $v_B = 4i + 3j$, and $v_C = 2i + 5j - 3k$, determine the angular momentum H_o of the system about O.

$\overline{H}_o = \sum \overline{r} \times m\overline{v}$

$m_A = 1$

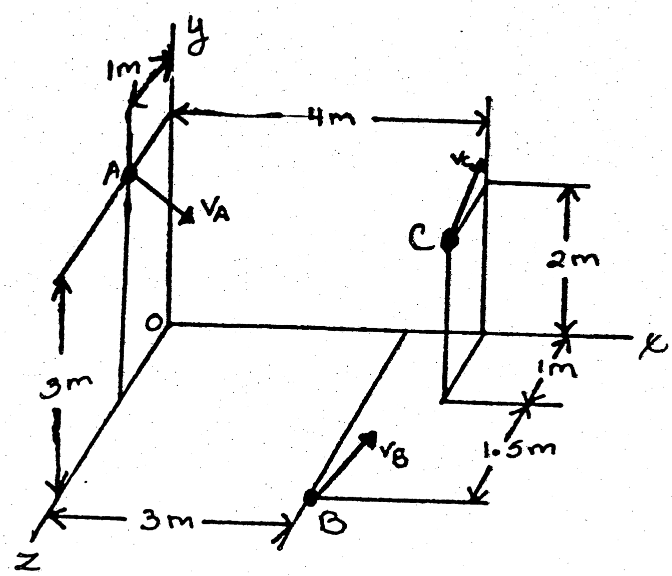
$m_B = 2$

$m_C = 3$

$\overline{v}_A = 3\overline{i} - 2\overline{j} + 4\overline{k}$

$\overline{v}_B = 4\overline{i} + 3\overline{j}$

$\overline{v}_C = 2\overline{i} + 5\overline{j} - 3\overline{k}$



$$\overline{H}_o = (1)\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & 3 & 1 \\ 3 & -2 & 4 \end{vmatrix} + (2)\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 3 & 0 & 2.5 \\ 4 & 3 & 0 \end{vmatrix} + (3)\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 4 & 2 & 1 \\ 2 & 5 & -3 \end{vmatrix}$$

$\overline{H}_o = \overline{i}\ (12+2) + (2)(-7.5) + (3)(-6-5)$

$-\overline{j}\ (0-3) + (2)(0-10) + (3)(-12-2)$

$+\overline{k}\ (0-9) + (2)(9-0) + (3)(20-4)$

$\overline{H}_o = (-34\ kg - m^2/sec)\overline{i} + (65\ kg - m^2/s)\overline{j} + (57\ kg - m^2/s)\overline{k}$

A 10 kg projectile is passing through the origin O with a velocity $v_o = 60i$ (m/s) when it explodes into two fragments, A and B, of mass 4 kg and 6 kg respectively. Knowing that 2 s later, the position of the first fragment is A (150 m, 12m, -24m) determine the position of fragment B at the same instant.

$m = 10$ kg $\qquad m_A = 4$ kg

$v_o = 60\bar{i}$ $\qquad m_B = 6$ kg

Motion of mass center is uniform in x & z and uniformly accelerated in y. Therefore @ t = 2s

$$\bar{r} = (v_o t)\bar{i} - (\tfrac{1}{2}gt^2)\bar{j}$$

$$\bar{r} = (60)(2)\bar{i} - (\tfrac{1}{2})(9.8)(2^2)\bar{j}$$

$$\bar{r} = 120\,\bar{i} - 19.6\,\bar{j}$$

Position of A @ t = 2s

$$\bar{r}_A = 150\bar{i} + 12\,\bar{j} - 24\,\bar{k}$$

$$m\bar{r} = m_A\bar{r}_A + m_B\bar{r}_B$$

$$(10)(120\bar{i} - 19.6\bar{j}) = (4)(150\bar{i} + 12\bar{j} - 24\bar{k}) + (6)(x_B\bar{i} + y_B\bar{j} + z_B\bar{k})$$

$\bar{i}$: $\quad 1200 = 600 + 6x_B$

$x_B = 100$

$\bar{j}$: $\quad -196 = 48 + 6y_B$

$y_B = -40.7$

$\bar{k}$: $\quad 0 = -96 + 6z_B$

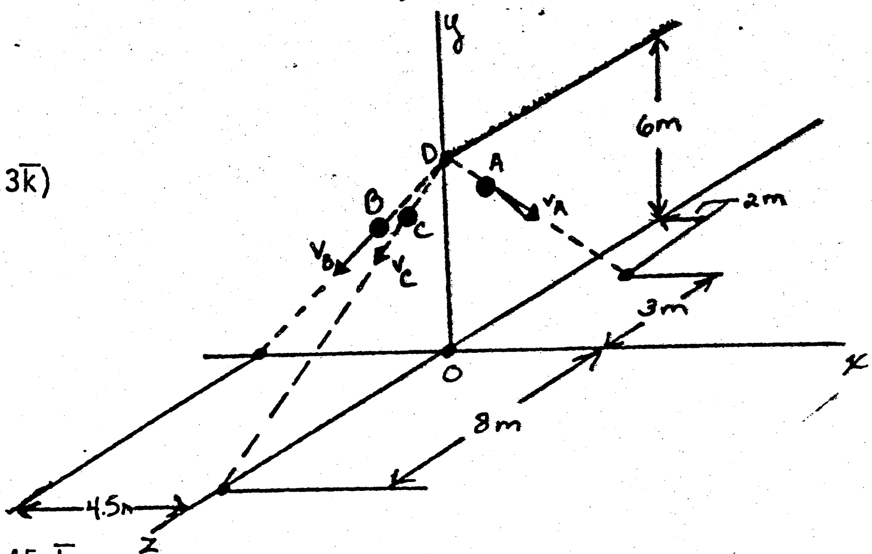
$z_B = 16$

Position of B @ t = 2s

(100m, -40.7m, 16m)

A 5 kg object is falling vertically when at point D, it explodes into three fragments A, B, and C, weighing, respectively, 1.5 kg, 2.5 kg and 1 kg. Immediately after the explosion the velocity of each fragment is directed as shosn and the speed of fragment A is observed to be 70 m/s. Determine the velocity of the 5 kg object immediately before the explosion.

DETERMINE UNIT VECTORS



$$\lambda_A = \frac{2\bar{i} - 6\bar{j} - 3k}{7} = \frac{1}{7}(2\bar{i} - 6\bar{j} - 3\bar{k})$$

$$\lambda_B = \frac{-4.5\bar{i} - 6\bar{j}}{7.5} = \frac{1}{5}(-3\bar{i} - 4\bar{j})$$

$$\lambda_C = \frac{-6\bar{j} + 8\bar{k}}{10} = \frac{1}{5}(-3\bar{j} + 4\bar{k})$$

$$m_A\bar{v}_A = 1.5(70\lambda_A) = 30\bar{i} - 90\,\bar{j} - 45\,\bar{k}$$

$$m_B\bar{v}_B = 2.5\,(v_B\lambda_B) = v_B(-1.5\bar{i} - 2\bar{j})$$

$$m_C\bar{v}_C = 1(v_C\lambda_C) = v_C(-.6\bar{j} + .8\bar{k})$$

$$m\bar{v}_o = 5(-v_o\bar{j}) = -5v_o\bar{j}$$

<u>CONSERVATION OF LINEAR MOMENTUM</u>

$$m\bar{v}_o = m_A\bar{v}_A + m_B\bar{v}_B + m_C\bar{v}_C$$

$\bar{i}$: $30 - 1.5\,v_B = 0$ $\qquad v_B = 20$ m/s

$\bar{j}$: $-90 - 2v_B - .6v_C = -5v_o$

$\bar{k}$: $-45 + .8\,v_C = 0$ $\qquad v_C = 56.25$ m/s

$$5v_o = 90 + 2(20) + (.6)(56.25)$$

$v_o = 32.75$ $\qquad \bar{v}_o = -v_o\bar{j}$

$$\bar{v}_o = (-32.75 \text{ m/s})\bar{j}$$

DYNAMICS

PROBLEN N 6/5

Derive the relation $\overline{H}_o = \overline{r} \times m\overline{v} + \overline{H}_G$ between the angular momenta H_o and H_G. The vectors r and v define, respectively, the position and velocity of the mass center G of the system of particles relative to the newtonian frame of reference Oxyz and m represents the total mass of the system.

$$\overline{H}_o = \sum(\overline{r}_i \times m_i\overline{v}_i)$$

$$\overline{H}_G = \sum(\overline{r}_i' \times m_i\overline{v}_i)$$

$$\overline{r}_i = \overline{r} + \overline{r}_i'$$

where $\overline{r}$ is vector from Oxyz reference frame to the center of mass reference frame

$$\overline{H}_o = \sum(\overline{r} + \overline{r}_i') \times m_i\overline{v}_i$$

$$= \sum(\overline{r} \times m_i\overline{v}_i) + \sum(\overline{r}_i' + m_i\overline{v}_i)$$

$$= \overline{r} \times \sum m_i\overline{v}_i + \sum(\overline{r}_i' \times m_i\overline{v}_i)$$

but $m_i\overline{v}_i = m\overline{v}$ and $\sum(\overline{r}_i' \times m_i\overline{v}_i) = \overline{H}_G$

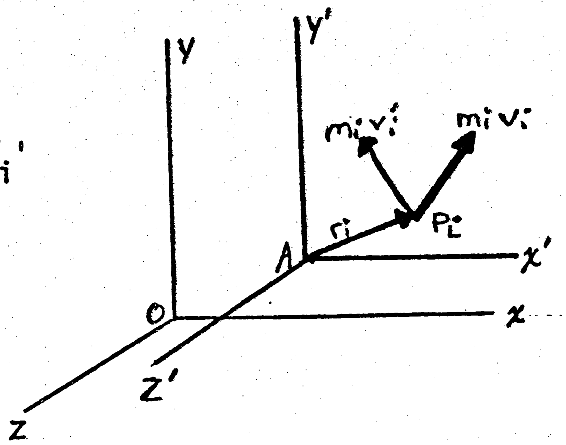
therefore:

$$\overline{H}_o = \overline{r} \times m\overline{v} + \overline{H}_G$$

Consider the frame of reference Ax'y'z' in translation with respect to the Newtonian frame of reference Oxyz. We define the angular momentum H_A' of a system of n particles about A as the sum

$$\overline{H}_A' = \sum_{i=1}^{n} \overline{r}_i' \times m_i\overline{v}_i'$$

of the moments about A of the momenta $m_i v_i'$ of the particles in their motion relative to the newtonian frame Oxyz, show that $H_A = H_A'$ at a given instand if and only if one of the following conditions is satisfied at that instant: (a) A has zero velocity with respect to the frame Oxyz, (b) A coincides with the mass center G of the system, (c) the velocity v_A relative to Oxyz is directed along the line AG.



$$\overline{v}_i = \overline{v}_A + \overline{v}_i'$$

$$\overline{H}_A = \sum \overline{r}_i' \times m_i\overline{v}_i = \sum \overline{r}_i' \times m_i\overline{v}_A + \sum \overline{r}_i' \times m_i\overline{v}_i'$$

$$= (\Sigma\, m_i\overline{r}_i') = m\overline{r}' = m\, AG$$

where G is the mass center of the system.

Therefore:

$$\overline{H}_A = mAG \times \overline{v}_A + H_A'$$

$\overline{H}_A = \overline{H}_A'$ if the cross product is zero

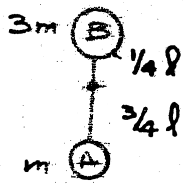
i.e.

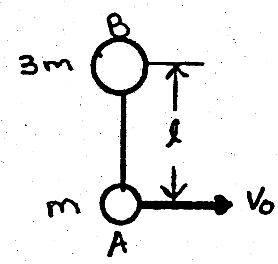
(a) $\overline{v}_A = 0$

(b) $\overrightarrow{AG} = 0$ (A & G coincide)

(c) $\overline{v}_A \parallel \overrightarrow{AG}$ ($\overline{v}_A$ is directed along AG)

Two small spheres A and B respectively of mass m and 3m are connected by a rigid rod of length l and negligible mass. The two spheres are resting on a horizontal, frictionless surface when A is suddenly given the velocity $\bar{v}_o = v_o\bar{i}$. Determine (a) the linear momentum of the system and its angular momentum about its mass center G, (b) the velocities of A and B after the rod AB has rotated through 90^o, (c) the velocities of A and B after the rod AB has rotated through 180^o.





(a) $\bar{L} = m_A\bar{v}_A + m_B\bar{v}_B = mv_o\bar{i}$

$\bar{L} = mv_o\bar{i}$

$\bar{H}_G = (-\frac{3}{4}\ \bar{j}) \times mv_o\bar{i}$

$\bar{H}_G = \frac{3}{4} m\ \ v_o\bar{k}$

<u>Motion of Mass Center</u>

since $\bar{L} = (4m)\bar{v}$ and $\bar{L} = mv_o\bar{i}$=const., we have for entire motion:

$(4m)\bar{v} = mv_o\bar{i}$

$\bar{v} = \frac{1}{4}v_o\bar{i}$ (1)

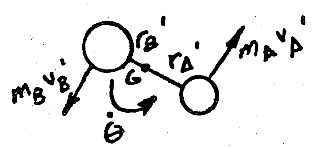
<u>Motion about Mass Center</u>

$\bar{H}_G = \bar{H}_G{}' = \ \bar{r}_i{}' \times m_i\bar{v}_i{}'$

$= \bar{r}_A{}' \times m\bar{v}_A{}' + \bar{r}_B{}' \times m\bar{v}_B{}'$

$r_A{}' = \frac{3}{4}l$

$r_B{}' = \frac{1}{4}l$



$m_A = m \qquad m_B = 3m$

$v_A{}' = (\frac{3}{4}l)\ \dot{\theta} \qquad v_B{}' = (\frac{1}{4}l)\dot{\theta}$

<u>Therefore</u>

$\bar{H}_G = (\frac{3}{4}l)m(\frac{3}{4}l)\dot{\theta}\ \bar{k} + (\frac{1}{4}l)(3m)(\frac{1}{4}l)\dot{\theta}\ \bar{k}$

$\bar{H}_G = (\frac{9}{16} + \frac{3}{16})m\ l^2\dot{\theta}\ \bar{k}$

$= \frac{3}{4}\ m\ l^2\dot{\theta}\ \bar{k}$

From Part (a):

$H_G = \frac{3}{4}\ m\ l\ v_o\bar{k}$ = const.

<u>Thus:</u>

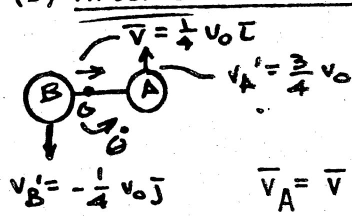
$\frac{3}{4}m\ l^2\dot{\theta} = \frac{3}{4}\ m\ l\ v_o$

$\dot{\theta} = \frac{v_o}{l}$ and,

$v_A{}' = \frac{3}{4}\ l\dot{\theta} = \frac{3}{4}\ \frac{v_o}{l} = \frac{3}{4}\ v_o$ (2)

$v_B{}' = \frac{1}{4}\ l\dot{\theta} = \frac{1}{4}l\ \frac{v_o}{l} = \frac{1}{4}\ v_o$ (3)

(b) <u>After 90^o Rotation</u>

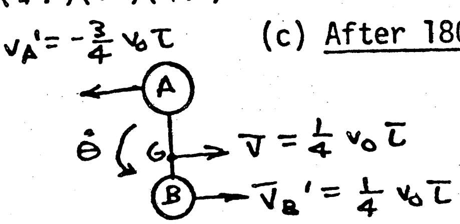


$\bar{v}_A = \bar{v} + \bar{v}_A{}' = \frac{1}{4}\ v_o\bar{i} + \frac{3}{4}v_o\bar{j}$

$v_B = \bar{v} + \bar{v}_B{}'$

$v_B = \frac{1}{4}v_o\bar{i} - \frac{1}{4}\ v_o\bar{j}$

(c) <u>After 180^o Rotation</u>



$\bar{v}_A = \bar{v} + \bar{v}_A{}' = \frac{1}{4}\ v_o\bar{i} - \frac{3}{4}\ v_o\bar{i} = -\frac{1}{2}\ v_o\bar{i}$

$\bar{v}_B = \bar{v} + \bar{v}_B{}' = \frac{1}{4}\ v_o\bar{i} + \frac{1}{4}v_o\bar{i} = \frac{1}{2}v_o\bar{i}$

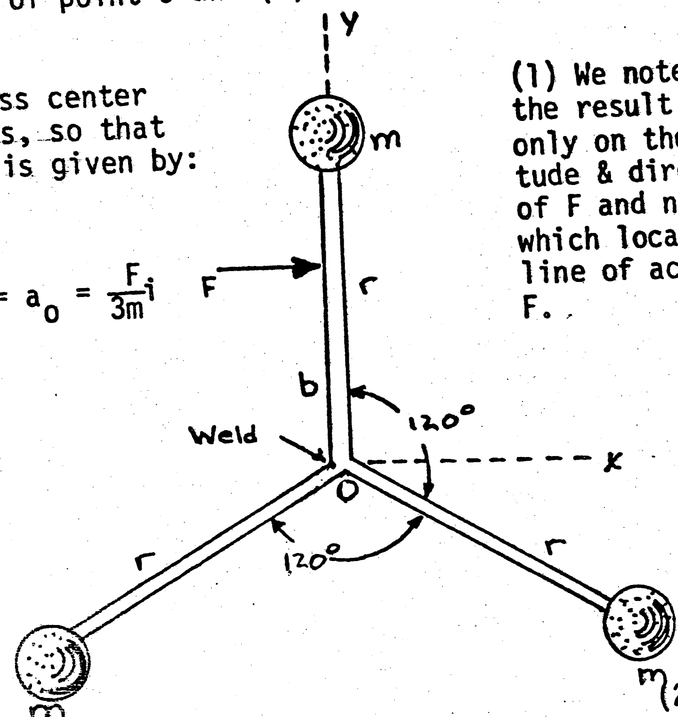
DYNAMICS

Each of the three balls has a mass m and is welded to the rigid equiangular frame of negligible mass. The assembly rests on a smooth horizontal surface. If a force F is suddenly applied to the one bar as shown, determine (a) the acceleration of point O and (b) the angular acceleration $\ddot{\theta}$ of the frame.

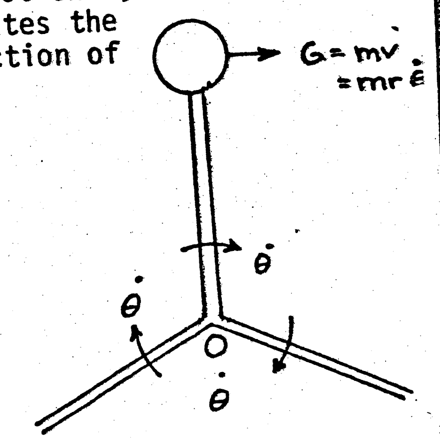
a) Point O is the mass center of the three balls, so that its acceleration is given by:

1) $F = m\bar{a}$

$Fi = 3m\bar{a} \quad \bar{a} = a_0 = \frac{F}{3m}i$



(1) We note that the result depends only on the magnitude & direction of F and not on b, which locates the line of action of F.



(2) Although $\dot{\theta}$ is initially zero we need the expression for Ho in order to get Ho. We observe also that $\dot{\theta}$ is independent of the motion of O.

(b) We determine $\ddot{\theta}$ from the moment principle. To find $\bar{H}$ we note that the velocity of each ball relative to the mass center O as measured in the nonrotating axes x-y is $r\dot{\theta}$ where $\dot{\theta}$ is the common angular velocity of the spokes. The angular momentum of the system about O is the sum of the moments of the relative linear momenta as shown by:

$$Ho = \bar{H} = 3(mr\dot{\theta})r = 3\,mr^2\dot{\theta}$$

(2) $\quad \bar{M} = \dot{\bar{H}} \quad Fb = \frac{d}{dt}(3mr^2\dot{\theta}) = 3mr^2\ddot{\theta}$

so

$$\ddot{\theta} = \frac{Fb}{3mr^2}$$

From a point 2 meters above the ground a ball is thrown upward at an angle of 60^o with the horizontal against a wall 7 meters distant. If its initial velocity is 20 meters/second and e for the ball is .50, where and with what velocity will the ball strike the ground?

$v_{ox} = 20 \cos 60 = 10$ m/s

$x = x_o + v_{ox}t$

At the wall x = 7

$7 = 0 + 10t \quad t = .7s$

$y = y_o + v_{oy}t - \frac{1}{2}gt^2$

$y = 2 + 20 \sin 60(.7) - 4.9(.7)^2$

$y = 11.72$ height where ball hits wall

$v_y = v_{oy} - gt$

$= 20 \sin 60 - 9.8(.7)$

$= 10.46$ m/s y component of velocity at wall

After collision

$v_x' = -.5\, v_x = -.5(10) = -5$

Projectile After Collision

$y = y_o + v_{oy}t - \frac{1}{2}gt^2$

$0 = 11.72 + 10.46t - 4.9t^2$

$$t = \frac{2.13 + \sqrt{4.54 + 9.56}}{2} = 2.95s$$

Location where it hits ground

$x = x_o + v_{ox}'\, t$

$= -5(2.95) = 14.73$m (to the left of wall)

Speed when hitting ground

$v_x = -5$ m/s (to the left)

$v_y = v_{oy} - gt$

$= 10.46 - 9.8\,(2.95)$

$= -18.42$ m/s (down)

$\tilde{v} = 5\hat{i} - 18.42\hat{j}$ $\quad\quad v = 19.08$ m/s