DYNAMICS PROBLEM N5/1

A 1000kg automobile is moving at a speed of 70 km/h when the brakes are fully applied, causing all four wheels to skid. Determine the time required to stop the automobile (a) on concrete ($\mu = .80$), (b) on ice ($\mu = .10$).

$$70 \text{ km/hr} = 19.4 \text{m/sec}$$

(a) $V'_A = 0$

$$m_A V_A - \int P dt = m_A V'_A = 0$$

Since the frictional force is constant, $\int P dt = \tilde{P} \Delta t$

$$m_A V_A - \tilde{P} \Delta t = 0$$

$f = P = \mu N$

$$N = mg$$

$$m_A V_A = \mu N \Delta t$$

$$\Delta t = \frac{1000(19.4)}{.8(1000)(9.8)}$$

$$\Delta t = 2.5 \text{ sec}$$

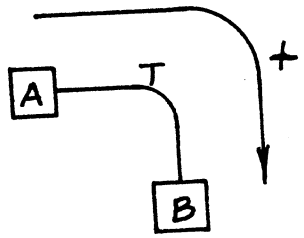
(b) On ice ($\mu = .1$)

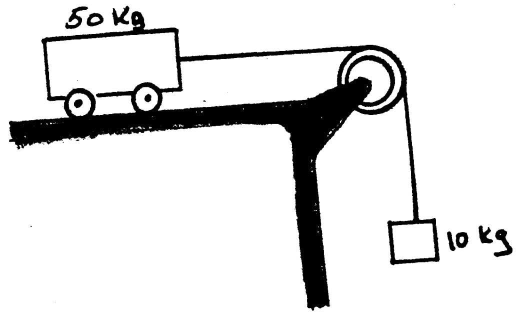
$$\Delta t = \frac{m_A V_A}{\mu m_A g}$$

$$\Delta t = \frac{V_A}{\mu g} = \frac{19.4}{.1(9.8)}$$

$$\Delta t = 19.8 \text{ sec}$$

The initial velocity of the 50 kg car is 5 m/s to the left. Determine the time t at which the car has (a) no velocity, (b) a velocity of 5 m/s to the right.





$$m_A v_A + \int T dt = m_A v_A' = 0$$

$$m_B v_B + \int (W_B - T) dt = m_B v_B' = 0$$

(a)

$$m_A v_A + \int T dt = 0$$

$$+ \quad m_B v_B - \int T dt + \int w_B dt = 0$$

$$m_A v_A + m_B v_B + \int w_B dt = 0$$

(b)

$$m_A v_A + \int T dt = m_A v_A'$$

$$m_B v_B - \int T dt + \int w_B dt = m_B v_B'$$

$$m_A v_A + m_B v_B + \int w_B dt = m_A v_A' + m_B v_B'$$

$$v_A = v_B; \quad v_A' = v_B'$$

(a)

~~$m_A v_A + m_B v_A + \int w_B dt = 0$~~

~~$60(-5) + 10(9.8)(t) = 0$~~

~~$t = 3.06$ sec~~

$\frac{300}{98} = 3.06$

$(50)(-5) + 10(9.8)(t) = 0$

$10(9.8)t) = +250$

time $= \frac{250}{98} = 2.55$ SECONDS

(b)

$(m_A + m_B)v_A + m_B g \quad t = (m_A + m_B)(v_A')$

$(60)(-5) + 9.8(10) \quad t = 60(5)$

$t = 6.1$ sec

$(50)(-5) + (9.8)(10)t = 50(5)$

$98t = 500$

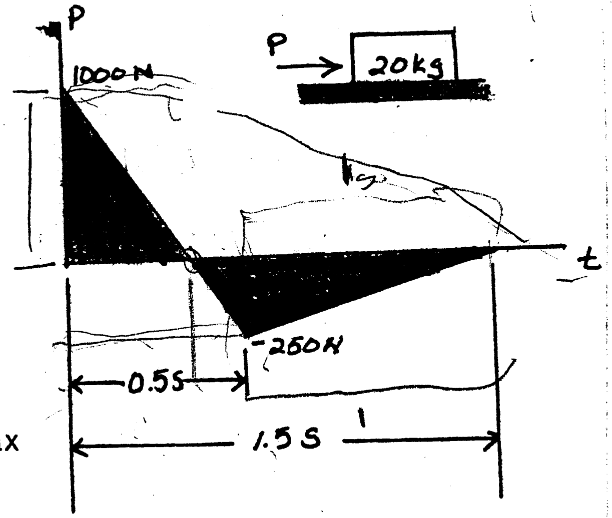
$t = 5.1$ SECONDS

Ck: $-250 + 500 = 250$

$500 = 500$

DYNAMICS PROBLEM N 5/3

A 20 kg block is initially at rest and is subjected to a force P which varies as shown. Neglecting the effect of friction, determine (a) the maximum speed attained by the block (b) the speed of the block at t = 1.5 s.



(a) Maximum speed will be reached when P becomes zero, i.e., at t = .4 sec.

$$mv_1 + \int Fdt = mv_2$$

$$0 + \int_0^{.4} Pdt = mv_2$$

$$\int_0^{.4} Pdt = \text{area under triangle} = mV_{max}$$

$$\frac{1}{2}(.4)(1000) = 20V_{max}$$

$$V_{max} = \frac{(.2)(1000)}{20}$$

$$V_{max} = 10 \text{ m/sec}$$

$\frac{1000}{x} = \frac{1250}{0.5}$

area under t

$x = .4$

(b) $$mv_1 + \int_0^{1.5} Pdt = 20v_2$$

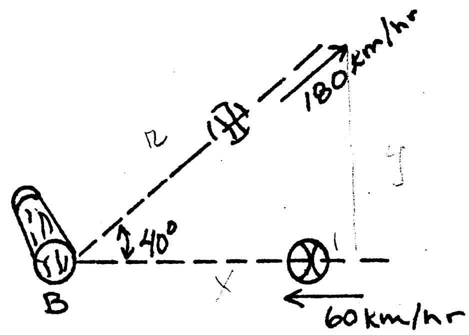
$$0 + \left[\frac{1}{2}(.4)(1000) + (\tfrac{1}{2})(1.1)(-250)\right] = 20v_2$$

$$v_2 = 3.13 \text{ m/s}$$

$\frac{1250}{0.5} = \frac{1000}{x}$

DYNAMICS PROBLEM N5/4

A 100 gm baseball is pitched with a velocity of 60 km/h toward a batter. After the ball is hit by the bat B, it has a velocity of 180km/h in the direction shown. If the bat and ball are in contact for 0.025s, determine the average impulsive force exerted on the bat during the impact.



$m = 100gm = .1kg$

$V_{1x} = 60km/hr = 16.7m/sec$

$V_{1y} = 0$

$mV_{1x} + \int F_x dt = mV_{2x}$

$mV_{1y} + \int F_y dt = mV_{2y}$

Positive to right

180 km/hr = 50m/s

$V_{2x} = 50 \cos 40^0$

$V_{2y} = 50 \sin 40^0$

$.1(-16.7) + F_x(.025) = .1(50 \cos 40)$

$.1(0) + F_y(.025) = .1(50 \sin 40)$

$F_x = 219.9N \qquad F_y = 128.6N$

$F = 254.7N$ @ 40^0 ~~X~~ → 30.32^0

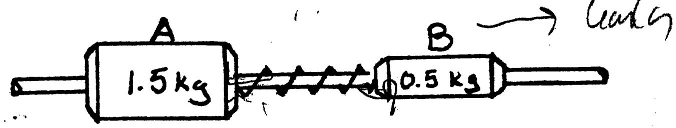
$\frac{F_y}{F_x} = \tan \Theta$

$\Theta = 30.32^0$

DYNAMICS PROBLEM N 5/5

Collars A and B are moved toward each other, thus compressing the spring and are released from rest. The spring is not attached to the collars. Neglecting the effect of friction, and knowing that colalr B is observed to move to the right with a velocity of 6 m/s, determine (a) the corresponding velocity of collar A, (b) the kinetic energy of each collar.

Total Momentum is Conserved



Positive to the right

Impulse

(a) $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$v_A = v_B = 0$

$0 = 1.5\,(v_A') + .5(6)$

$v_A' = \dfrac{-(.5)(6.0)}{1.5}$

$v_A' = -2\text{m/s}$

$v_A' = 2$ m/s to the left

(b) $E_A = \frac{1}{2} m_A v_A'^2 = \frac{1}{2}(1.5)(2)^2$

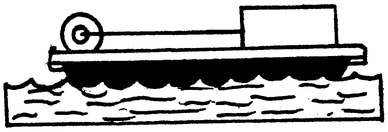
$E_A = 3$ N.m = 3J

$E_B = \frac{1}{2} m_B v_B'^2 = \frac{1}{2}(.5)(6)^2$

$E_B = 9\text{N} \cdot \text{m} = 9\text{J}$

A barge is initially at rest and carries a 600 kg crate. The barge has a mass of 3000 kg and is equipped with a winch which is used to move the crate along the deck. Neglecting any friction between the crate and the barge, determine (a) the velocity of both the barge and the crate when the winch is drawing the rop at the rate of 1.5 m/s, (b) the final position of the barge after 12 m of rope has been drawn in by the winch, (c) Solve parts (a) and (b) assuming that u = .30 between the crate and the barge.

Total momentum of barge and crate is conserved.



(a) $m_B v_B + m_C v_C = m_B v_B' + m_C v_C'$

$0 = m_B v_B' + m_C(v_B' - 1.5)$

$v_C' = v_B' - 1.5$

$v_B' = \dfrac{1.5\ m_C}{m_B + m_C} = \dfrac{1.5(600)}{3000+600}$

$v_B' = .25$ m/s

$v_C' = -1.25$ m/s

$v_C' = 1.25$ m/s

(b) Motion of Crate/barge

$X_{C/B} = vt$

$12\ m = 1.5\ t$

$t = 8s$

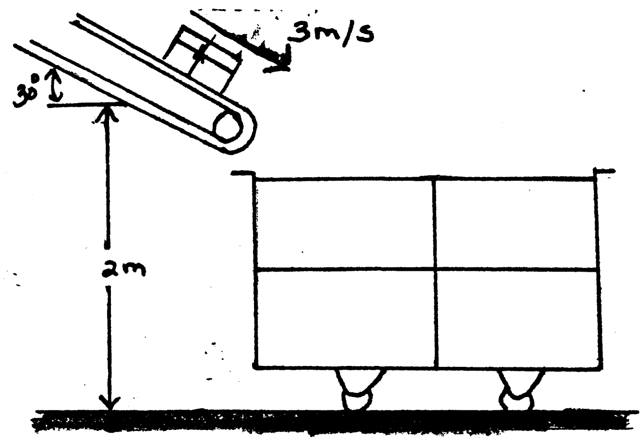
Barge

$x_B = v_B t = .25(8)$

$x_B = 2m$

(c) Since friction forces exerted on each other by the barge and the crate are internal forces, the answers to parts (a) & (b) are unchanged.

A 10 kg package is discharged from a conveyor belt with a velocity of 3 m/s and lands in a 25 kg cart. Knowing that the cart is initially at rest and may roll freely, determine the final velocity of the cart.



Collision is perfectly plastic

$m_A v_A + \not{m_B}^{\,0} v_B = (m_A + m_B)v'$

$v_A = v_o \cos \not{30^0}$

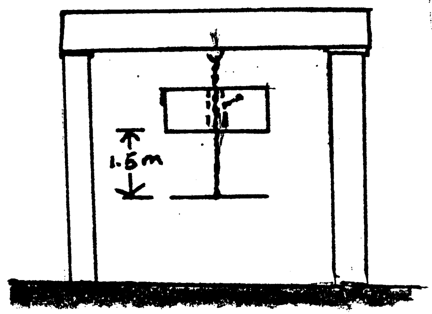
$v_A = 3 \cos 30^0$

(The x component of v_o causes motion in the horizontal direction)

$10(3 \cos 30^0) = 35\ (v')$

$v' = .74$ m/s

In order to test the resistance of a chain to impact, the chain is suspended from a 100 kg block supported by two columns. A rod attached to the last link of the chain is then hit by a 25 kg cylinder dropped from a 1.5 m height. Determine the initial impulse exerted on the chain assuming that the impact is perfectly plastic and that the columns supporting the dead weight (a) are perfectly rigid, (b) are equivalent to two perfectly elastic springs. (c) Determine the energy absorbed by the chain in parts (a) and (b).



(a) Perfectly rigid columns

velocity of block as it hits rod:

$E_1 = E_2$

$\not{m}g\, h_1 = \frac{1}{2}\not{m}v_2^2$

$(9.8)(1.5) = \frac{1}{2}(v_2^2)$

$v_2 = 5.4$ m/s

$\not{m}\overset{0}{\not{v_1}} - F\Delta t = mv_2$

$-F\Delta t = 25(-5.4)$

$F\Delta t = 135.5$ N·s

(b) Momentum for dead weight and block is conserved.

$m_1 v_1 = (m_{DW} + m_B)v_2$

$v_2 = \frac{m_1 v_1}{m_{DW} + m_B} = \frac{25(5.4)}{125}$

$v_2 = 1.08$ m/s

impulse for block

$mv_1 - F\Delta t = mv_2$

$(25)(5.4) - F\Delta t = 25(1.08)$

$F\Delta t = 108$ N · s

(c) Energy Absorbed

(a) $E = mgh = 25(9.8)(1.5)$

$E = 367.5$J

(b) $E_1 = E_2 + {}_1L_2$

${}_1L_2 = E_1 - E_2$

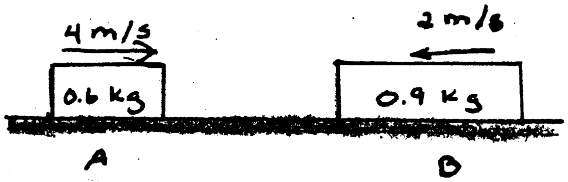
${}_1L_2 = \frac{1}{2}m_B v_1^2 - \frac{1}{2}(m_B + m_{DW})v_2^2$

${}_1L_2 = \frac{1}{2}(2$

${}_1L_2 = 29$

Energy a

Two steel blocks slide without friction on a horizontal surface; immediately before impact their velocities are as shown. Knowing that e = .75, determine (a) their velocities after impact, (b) the energy loss during impact.



Positive to right

(a) $m_A v_A + m_B v_B = m_B v_B' + m_A v_A'$

$.6(4) + .9(-2) = .6(v_A') + .9v_B'$

$v_A' + 1.5v_B' = 1$

$$e = \frac{v_B' - v_A'}{v_A - v_B}$$

$.75\left[4-(-2)\right] = v_B' - v_A'$

$v_B' = 4.5 + v_A'$

$v_A' + 1.5(4.5 + v_A') = 1$

$v_A' = -2.3$

$v_A' = 2.3$ m/s

$v_B' = 2.2$ m/s

(b) (1)→Before Collision

(2)→After Collision

$E_1 = E_2 + {}_1L_2$

${}_1L_2 = E_1 - E_2$

$${}_1L_2 = (\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2) - (\tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2)$$

$${}_1L_2 = \left[\tfrac{1}{2}(.6)(4^2) + \tfrac{1}{2}(.9)(-2)^2\right] - \left[\tfrac{1}{2}(.6)(-2.3)^2 + \tfrac{1}{2}(.9)(2.2)^2\right]$$

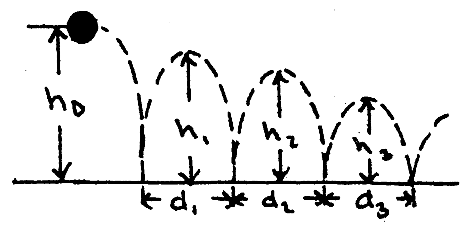
${}_1L_2 = 2.84$ J

A ball is dropped onto a frictionless floor and allowed to bounce several times as shown. Derive an expression for the coefficient of restitution in terms (a) of the heights of two successive bounces h_n and h_{n+1}, (b) of the lengths of two successive bounces d_n and d_{n+1}, (c) of the durations of two successive bounces t_n and t_{n+1}.

V_x; V_y → Initial horizontal and vertical velocities.

V'_x; V'_y → Horizontal and vertical after first bounce.

V''_x; V''_y → Horizontal and vertical after second bounce.



m

$$V'_y = eV_y$$

$$V_x = V'_x = V''_x$$

(a) Vertical Motion

$$mgh_o = \frac{1}{2} mV_y^2$$

$$V_y = \sqrt{2gh_o}$$

$$V'_y = \sqrt{2gh_1}$$

$$V'_y = eV_y$$

$$e\sqrt{2gh_o} = \sqrt{2gh_1}$$

$$h_1 = e^2 h_o$$

Result can be expanded to any two successive bounces.

$$h_{n+1} = e^2 h_n$$

$$e = \sqrt{\frac{h_{n+1}}{h_n}}$$

(b) t_1 → Time for first bounce
t_2 → Time for second bounce

$$V'_y = g(\frac{1}{2} t_1)$$

$$V''_y = g(\frac{1}{2} t_2)$$

$$V''_y = eV'_y$$

$$\therefore t_2 = et_1$$

$$t_{n+1} = et_n$$

Horizontal motion

$$d_1 = V'_x t_1 = V_x t_1$$

$$d_2 = V''_x t_2 = V_x t_2$$

$$\frac{d_x}{d_{x+1}} = \frac{t_2}{t_1} = e$$

$$d_2 = ed_1$$

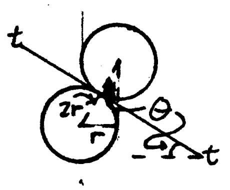
$$d_{n+1} = ed_n$$

$$e = \frac{d_{n+1}}{d_n}$$

(c) $t_{n+1} = et_n$

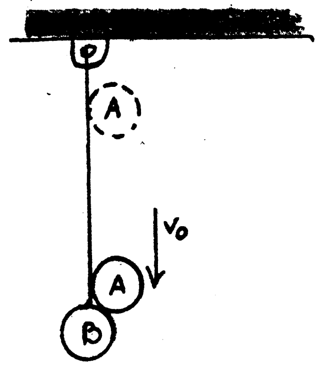
$$e = \frac{t_{n+1}}{t_n}$$

A ball is suspended by an inextensible cord. An identical ball A is released from rest when it is just touching the cord and acquires a velocity v_o before striking ball B. Assuming e = 1 and no friction, determine the velocity of each ball immediately after impact.



$\sin\theta = \frac{r}{2r}$ $\quad \sin\theta = \frac{r}{2r}$

$\theta = 30°$ $\quad \theta = 30^0$



$(V_A)_t = -v_o \cos 30^0$

$(V_A)_t = \frac{1}{2} v_o$

Total momentum conserved in horizontal direction

$$\overset{0}{\cancel{m_A v_A}} + \overset{0}{\cancel{m_B v_B}} = m_A v_A' + m_B v_B'$$

$$0 = m(v_A')_t \cos 30 + m(v_A')_n \sin 30 - m v_B'$$

$$.5\,(v_A')_n - v_B' + (\tfrac{1}{2} v_o) \cos 30 = 0$$

$$.5(v_A')_n - v_B' + 4.33\, v_o = 0 \qquad (1)$$

Relative velocities in normal direction

since e = 1

$$(v_A')_n - (v_B')_n = (v_B)_n - (v_A)_n$$

$$(v_A')_n + v_B' \sin 30^0 = v_o \cos 30^0$$

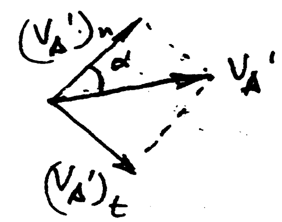
$$(v_A')_n = .866\, v_o - 0.5\, v_B' \qquad (2)$$

Substituting for $(v_A')_n$ from (2) into (1):

$$.433 v_o - .25 v_B' - v_B' + .433 v_o = 0$$

$$v_B' = 0.693\, v_o$$

$$(v_A')_n = .52 v_o$$



$$\tan = \frac{.5 v_o}{.52 v_o}$$

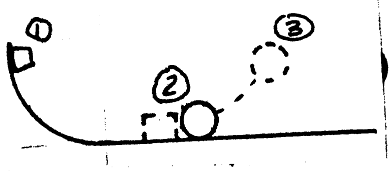
$$= 43.9^0$$

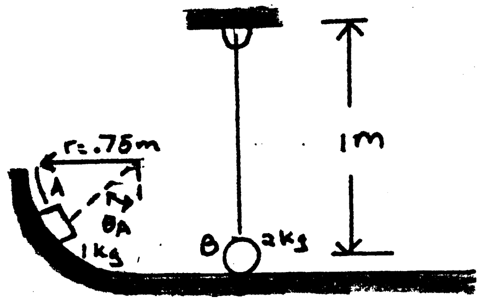
$$v_A' = .721\, v_o$$

Angle of v_A' w/ horiz. = 60^0 -

= 16.1^0

Block A is released when $\theta_A = 90^0$ and slides without friction until it strikes ball B. Knowing that e = .90, determine (a) the velocity of B immediately after impact, (b) the maximum tension in the cord holding B, (c) the maximum height to which ball B will rise.





Velocity of A before impact

$$mgh_1 = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gh}$$

$$v_2 = 3.8 \text{ m/s}$$

(a) $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$$1(3.8) + 0 = 1v_A' + 2v_B'$$

$$.90 = \frac{v_B' - v_A'}{v_A - v_B}$$

$$.9(3.8) = v_B' - v_A'$$

$$v_B' = 3.42 + v_A'$$

$$3.8 = v_A' + 2(3.42 + v_A')$$

$$v_A' = -1.01 \text{ m/s}$$

$$v_B' = 2.41 \text{ m/s}$$

(b) Max Tension occurs at lowest position of B

$$T - mg = ma_n$$

$$T = m(g + \frac{v^2}{\rho})$$

$$T = 2(9.8 + \frac{(2.41)^2}{1})$$

$$T = 31.2 \text{ N}$$

(c) Max height of B

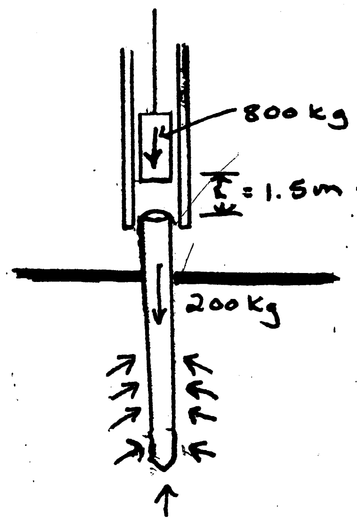
$$\frac{1}{2} mv_2^2 = mgh_3$$

$$h_3 = \frac{v_2^2}{2g}$$

$$h_3 = \frac{(2.41)^2}{2(9.8)}$$

$$h_3 = .3 \text{ m}$$

It is desired to drive the 200 kg pile into the ground until the resistance to its penetration is 11760 N. Each blow of the 800 kg hammer is the result of a 1.5 m free fall onto the top of the pile. Determine how far the pile will be driven into the ground when the 11760 N resistance is achieved. Assume that the impact is perfectly plastic.



Velocity of Hammer

$$mgh = (1/2)mv^2$$

$$v = \sqrt{2gh}$$

$$v_H = 5.4 \text{ m/s}$$

Impact (e=0)

Total momentum is conserved

$$m_H v_H + m_P v_P = (m_H + m_P)v'$$

$$v' = \frac{m_H v_H + m_P v_P^{\nearrow 0}}{(m_H + m_P)}$$

$$v' = \frac{800(5.4)}{800+200}$$

$$v' = 4.32 \text{ m/s}$$

Hammer and Pile move against ground resistance

$$E_1 + {}_1W_2 = E_2 + {}_1L_2$$

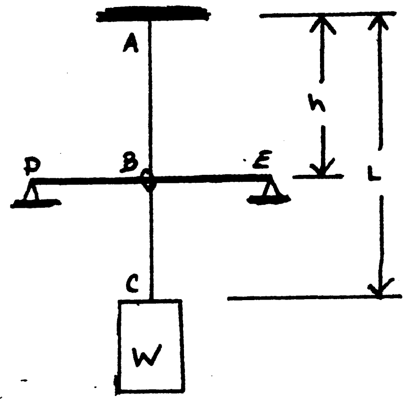
$$\frac{1}{2}(m_H + m_P)v'^2 + (m_H + m_P)gd = 0 + R(d)$$

$$\frac{1}{2}(800+200)(4.32)^2 + 9.8(800+200)d = 11760\ d$$

$$1960\ d = 9331.2$$

$$d = 4.76 \text{ m}$$

Two portions AB and BC of the same elastic cord are connected as shown. The portion of cord BC supports a load W while, initially, the portion AB is under no tension. Determine the maximum tension which will develop in the entire cord after the stick DE is suddenly broken.



For cord of length L, cross-sectional area A, and modules of elasticity E, tension F for elongation δ is:

$$F = \frac{AE}{L}\delta$$

<u>Before Break</u>: $W = \frac{AE}{L-h}\delta$ (1)

<u>Immediately After Break</u>:

$$F_o = \frac{AE}{L}\delta \qquad (2)$$

Since δ is same before and immediately after break, we eliminate δ from (1) & (2)

$$F_o = \frac{L-h}{L} W \qquad (3)$$

Let δ be max. deflection following break:

work of weight = mg δ

work of elastic force =

$$-\tfrac{1}{2}(F_o + F_{max})\,\delta$$

Since load is at rest immediately after break, and again at max. deflection:

$$E_1 = E_2 = 0$$

Thus, $E_1 + {}_1W_2 = E_2$

reduces to ${}_1W_2 = 0$

$${}_1W_2 = mg\delta - \tfrac{1}{2}(F_o + F_{max})\,\delta = 0$$

$$F_{max} = 2\,mg - F_o$$

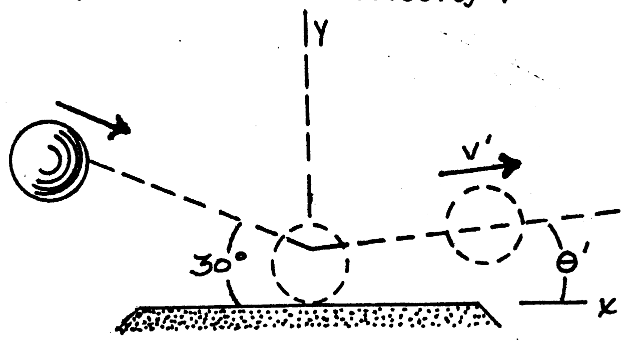
Recalling (3)

$$F_{max} = 2mg - \frac{L-h}{L}\,mg$$

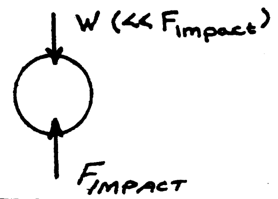
$$= mg\left(2 - 1 + \frac{h}{L}\right)$$

$$F_{max} = mg\left(1 + \frac{h}{L}\right)$$

A steel ball is projected onto the heavy metal plate with a velocity of 17m/sec at the 30 angle shown. If the coefficient of restitution between the ball and the plate is 0.5, compute the rebound velocity v' and its angle θ'.



The mass of the heavy plate may be considered infinite and its corresponding velocity zero after impact. The coefficient of restitution is applied to the velocity components normal to the plate in the direction of the impact force and gives:



$$e = \frac{\text{rel. vel. separation}}{\text{rel. vel. approach}},$$

$$0.5 = \frac{v' \sin\theta' - 0}{17 \sin 30^\circ + 0}, \quad v' \sin\theta' = 4.25 \text{ m/sec}$$

Momentum of the ball in the x-direction is unchanged since, with assumed smooth surfaces, there is no force acting on the ball in that direction. Thus:

$$m(17 \cos 30^0) = m(v' \cos\theta') \quad \text{or} \quad v' \cos\theta' = 14.7 \text{ m/sec}$$

Solution of the two equations gives $\tan\theta' = 4.25/14.7; \theta' = 16.1^0$

and $v' = 4.25/\sin 16.1^0 = 15.3$ m/sec