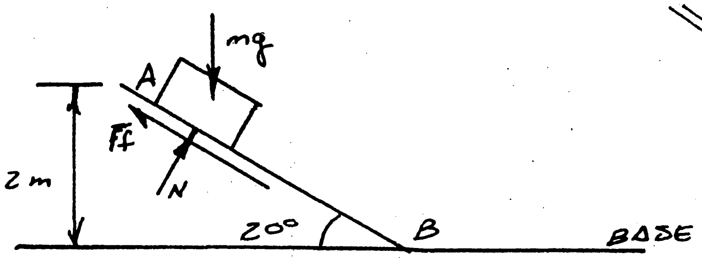
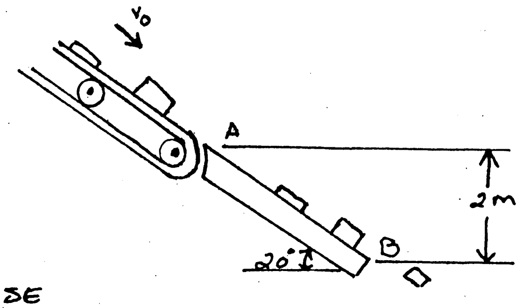
A conveyor belt moves at a constant speed v_0 and discharges packages onto a chute. The coefficient of friction between the packages and chute is .50. Knowing that the packages reach point B with a speed of 4 m/s, determine the speed v_0 of the conveyor belt.





$$E_A + {}_AW_B = E_B + {}_AL_B \qquad d = \frac{2}{\sin 20}$$

$$\tfrac{1}{2} m v_A^2 + mgh_A = \tfrac{1}{2} m v_B^2 + F_f d \qquad \sum Fy = N - mg\cos 20 = 0$$

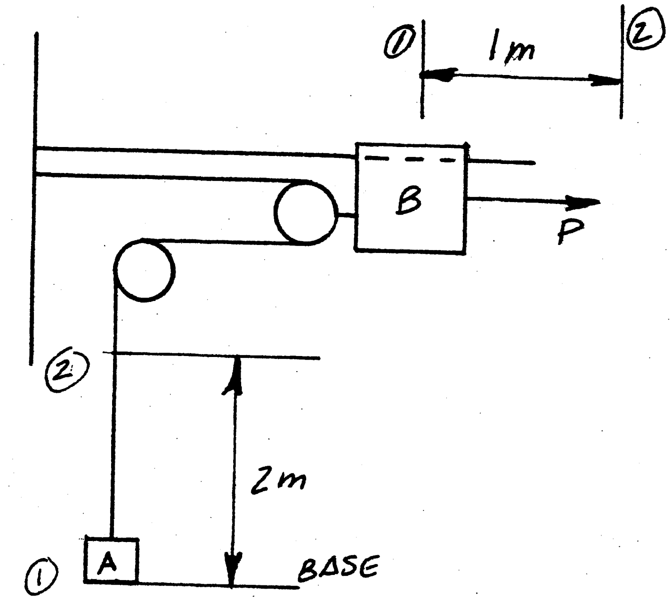
$$N = mg\cos 20$$

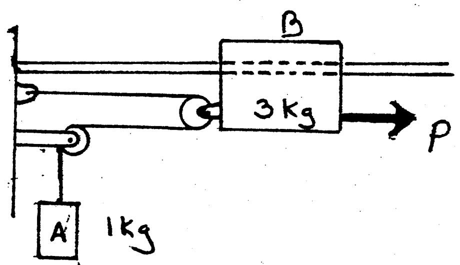
$$\tfrac{1}{2} \cancel{m} v_A^2 + \cancel{m}gh_A = \tfrac{1}{2}\cancel{m} v_B^2 + \mu \cancel{m}g \cos 20 \frac{2}{\sin 20} \qquad F_f = \mu mg\cos 20$$

$$v_A^2 = 2\left[\frac{v_B^2}{2} + 2\mu g \cot 20^{\circ} - g\,h_A\right]$$

$$v_A = 5.5 \text{ m/s}$$

Knowing that the system shown is initially at rest and neglecting the effect of friction, determine the force P required if the velocity of collar B is to be 3m/s after it has moved 1 m to the right.





$v_B = 3m/s$

$2x_B + x_A = C$

$2v_B + v_A = 0$

$V_B = \frac{V_A}{2}$

$v_A = 6m/s$

$m_A = 1\ kg$

$m_B = 3\ kg$

$$\cancel{E_1}^{0} + {}_1W_2 = E_2 + \cancel{{}_1L_2}^{0}$$

$$0 + (1)P = m_A g h_A + \tfrac{1}{2}(m_A v_A^2) + \tfrac{1}{2} m_B v_B^2$$

$$P = (1)(9.8)(2) + \tfrac{1}{2}(1)(6^2) + \tfrac{1}{2}(3)(3)^2$$

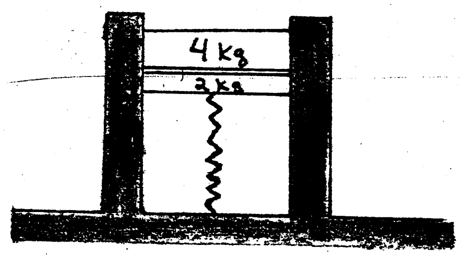
$$P = 51.1N$$

DYNAMICS

PROBLEM N 4/3

A 2 kg block is at rest on a spring of constant 400 N/m. A 4 kg block is held above the 2 kg block so that it just touches it, and released. Determine (a) the maximum velocity attained by the blocks, (b) the maximum force exerted on the blocks.

$E_1 = 0,\ E_2 = \frac{1}{2}mv^2 = \frac{1}{2}(6)v^2 = 3v^2$



$$F_{SPRING} = F_o + kx$$

$$= 2(9.81) + 400\ x$$

$$m_T g = 6(9.81) = 58.86\text{N}$$

$$_1W_2 = Wx - \int_0^x F_{spring}\, dx$$

$$= 58.86\ x - \int_0^x (19.62 + 400x)dx$$

$$= 58.86x - 19.62x - 200x^2$$

$$E_1 + {_1W_2} = E_2 \longrightarrow 39.24x - 200x^2 = 3v^2$$

(a) <u>MAX. VELOCITY:</u> V is max. when $(39.24x - 200x^2)$ is max.

i.e. when $\frac{d}{dx}(39.24x - 200x^2) = 0$

$39.24 - 400x = 0$

$x = 0.981$ m

$3v^2 = 39.24\ (.0981) - 200\ (0.0981)^2$

$v^2 = 0.6416$

$v = .801$ m/s

(b) Max force F_{SP} is obtained for x_{max}, i.e., when $v = 0$

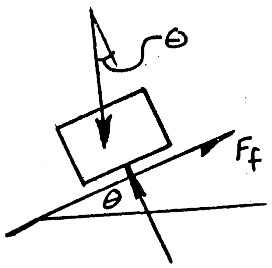
$39.24x - 200x^2 = 0$

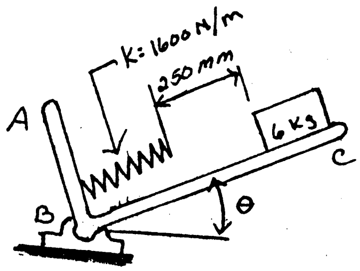
$x = 0.1962$ m

$F_{max} = 19.62 + 400(0.1962)$

$F_{max} = 98.1$ N

As the bracket ABC is slowly rotated, the 6 kg block starts to slide toward the spring when $\theta = 15^{0}$. The maximum deflection of the spring is observed to be 50 mm. Determine the values of the coefficients of static and kinetic friction.





STATIC FRICTION

$\mu_s = \tan\theta$

$\mu_s = \tan 15^{0}$

$\mu_s = .268$

$\sum F_y = N - mg\cos 15^{0} = 0$

$N = mg\cos 15^{0}$

KINETIC FRICTION

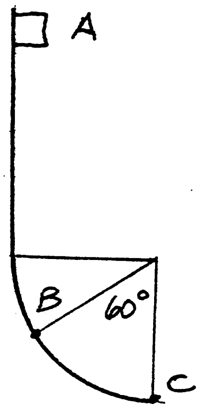
$E_1 + {}_1W_2 = E_2 + {}_1L_2$

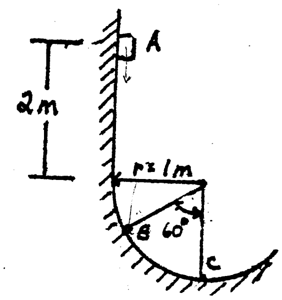
$0 + (mg \sin 15)(.25 + \overset{max}{.05}) = \tfrac{1}{2}ke^2 + F_f d$

$mg \sin 15(.3) = \tfrac{1}{2}(1600)(.05)^2 + \mu_K mg \cos 15^{0}(.25 + .05)$

$\mu_K = .151$

A .25 kg pellet is released from rest at A and slides without friction along the surface shown. Determine the force exerted by the surface on the pellet as it passes (a) point B, (b) point C.





$$\cancel{E_A}^{0} + {}_AW_B = E_B + \cancel{{}_AL_B}^{0}$$

$$.25(9.8)(2+.5) = (1/2)mv_B^2$$

$$v_B^2 = 49$$

$$E_B + {}_BW_C = E_C + \cancel{{}_BL_C}^{0}$$

$$(1/2)\cancel{m}v_B^2 + \cancel{m}gh = (1/2)\cancel{m}v_C^2$$

$$(49)/2 + .5(9.8) = (v_C^2)/2$$

$$v_C^2 = 58.8$$

Force at B

$$F_N = N - mg\cos 60 = ma_N$$

$$N = mg\cos 60 + mv_B^2/r$$

$$N = .25(9.8)\cos 60 + .25(49/1)$$

$$N_B = 13.5\ \text{N}$$

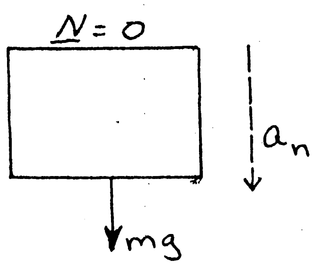
Force at C

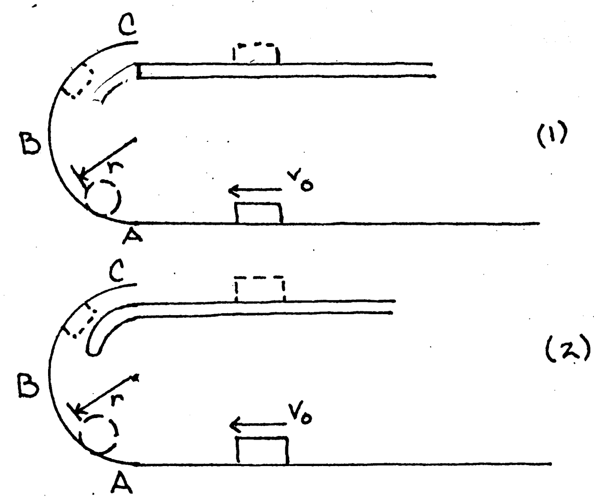
$$F_N = N - mg = m(v_C^2/r)$$

$$N = mg + mv_C^2/r$$

$$N_C = 17.2\ \text{N}$$

A small package of mass m is profected into a vertical return loop at A with a velocity v_o. The package travels without friction along a circle of radius r and is deposited on a horizontal surface at C. For each of the two loops shown, determine (a) the smallest velocity v_o for which the package will reach the horizontal surface at C, (b) the corresponding force exerted by the loop on the package as it passes point B.





At C $\underline{N} = 0$

$$\Sigma F_n = ma_n = mg$$

$$mg = m\frac{v_C^2}{r}$$

$$v_C^2 = gr$$

(1)

(a) $E_A + {}_AW_C = E_C + {}_AL_C$

$$\frac{1}{2}mv_o^2 = \frac{1}{2}mv_C^2 + mgh_A$$

$$h_A = 2r$$

$$\cancel{m}\frac{v_o^2}{2} = \frac{1}{2}\cancel{m}(gr) + \cancel{m}g2r$$

$$v_o^2 = gr + 4gr$$

$$v_o = \sqrt{5gr}$$

(b) $E_A + \cancel{{}_AW_B} = \cancel{E_B} + {}_AL_B$

$$\frac{1}{2}mv_o^2 = \frac{1}{2}mv_B^2 + mgh_B$$

$$h_B = r$$

$$\frac{1}{2}\cancel{m}(5gr) = \frac{1}{2}\cancel{m}v_B^2 + \cancel{m}gr$$

$$5gr = v_B^2 + 2gr$$

$$v_B^2 = 3gr$$

$$\Sigma F_n = N_B = ma_n$$

$$N_B = m\frac{v_B^2}{r}$$

$$N_B = m\frac{3gr}{r}$$

$$N_B = 3mg$$

(2)

At C $v_c = 0$

(a) $E_A + {}_A\cancel{W}_C^{\nearrow 0} = E_C + {}_A\cancel{L}_C^{\nearrow 0}$

$$\frac{1}{2} mv_o^2 = mgh_C$$

$$h_C = 2r$$

$$\frac{1}{2} \cancel{m}v_o^2 = \cancel{m}g(2r)$$

$$v_o^2 = 4gr$$

$$v_o = 2\sqrt{gr}$$

(b) $E_A = E_B$

$$\frac{1}{2} \cancel{m}v_o^2 = \frac{1}{2} \cancel{m}v_B^2 + \cancel{m}gh_B$$

$$h_B = r$$

$$\frac{1}{2} (4gr) = \frac{1}{2} v_B^2 + gr$$

$$4gr = v_B^2 + 2gr$$

$$v_B^2 = 2gr$$

$$\Sigma F_n = N = ma_n$$

$$N = m \frac{v_B^2}{r}$$

$$N = m \frac{2gr}{r}$$

$$N = 2mg$$

A 1500 kg automobile travels 200 m while being accelerated at a uniform rate from 50 to 75 km/hr. During the entire motion, the automobile is traveling on a horizontal road, and the rolling resistance is equal to 2 percent of the weight of the automobile. Determine (a) the maximum power required, (b) the power required to maintain a constant speed of 75 km/hr.

$v_f^2 = v_o^2 + 2as$

50 km/hr = 13.9 m/s

75 km/hr = 20.8 m/s

m = 1500

$20.8^2 = 13.9^2 + 2(a)(200)$

$a = \frac{20.8^2 - 13.9^2}{400}$

$a = .6 \text{ m/sec}^2$

$F_p \rightarrow$ PROPULSIVE FORCE

$\Sigma F = ma$

$\Sigma F = F_p - .02mg = ma$

$F_p = .02mg + ma$

$F_p = .02(1500)(9.8) + 1500(.6)$

$F_p = 1194$

(a) MAX POWER

$V_{max} = 20.8 \text{ m/s}$

$P = FV$

$P = 1194\ (20.8)$

$P = 25 \times 10^3 \text{ N} \cdot \text{m/s}$

$P = 25 \text{ kW}$

(b) CONST. SPEED

$\Sigma F = 0$

$F_p - .02 \text{ mg} = 0$

$F_p = .02 \text{ mg}$

$F_p = 294.3$

$P = F_v = (294.3)(20.8)$

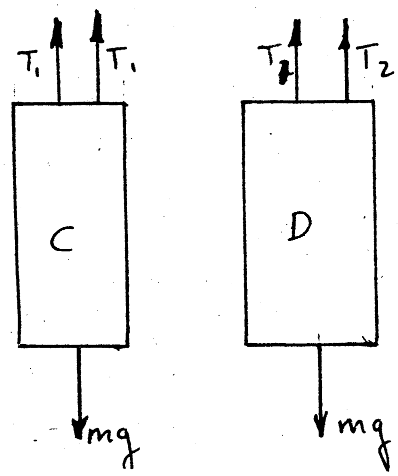
$P = 6.13 \times 10^3 \text{N} \cdot \text{m/s}$

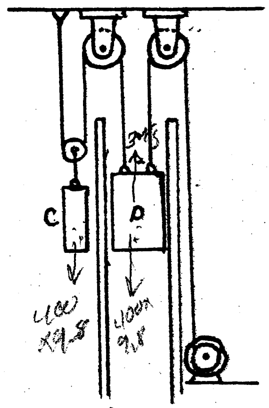
$P = 6.13 \text{ kW}$

DYNAMICS

PROBLEM N 4/8

The dumbwaiter D and its couterweight C weigh 400 kg each. Determine the power required when the dumbwaiter is moving upward at a constant speed of 3 m/s, (b)has an instantaneous velocity of 3 m/s upward and an upward acceleration of 1 m/s^2.





$v_P = v_{motor}$

$T_1 = \frac{400 \times 9.8}{2}$

$v_P = \frac{v_C}{2}$

$P = F \cdot v$

$\cancel{400 \times 9.8}$

$\times 2 \quad 2T_1 = 400 \times 9.8$

$T = \cancel{200 \times 9.8}$

$200 \times 9.8 - 400 \times 9.8 \times$

$P =$

(a) C & D ARE IN EQUILIBRIUM

$\therefore T_1 = T_2$

$T_1 = 1960$ N

$T_2 = 1960$ N

$P = T_2 V = 1960\ (3)$

$P = 5880$ N $\cdot$ m/sec

(b) $a_c = .5$ m/s^2

COUNTERWEIGHT

$\Sigma F_y = 2T_1 - mg = m(a_c)$

$2T_1 - 3920 = 400(.5)$

$T_1 = 1860$

DUMBWAITER

$a_D = 1$ m/s^2

$\Sigma F_y = T_1 + T_2 - mg = ma_D$

$1860 + T_2 = m\ (g + a_D)$

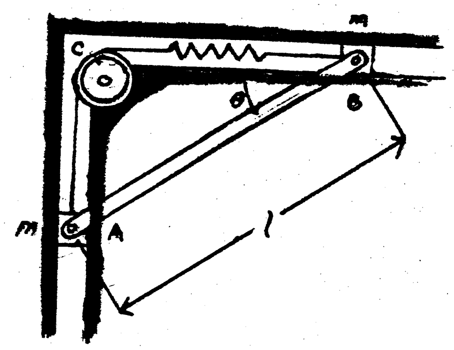
$1860 + T_2 = 400\ (10.8)$

$T_2 = 2460$

Power $= T_2 V = (2460)(3)$

Power $= 7380$ N $\cdot$ m/sec

A slender rod AB of negligible mass is attached to blocks A and B, each of mass m. The constant of the spring is k and the spring is undeformed when AB is horizontal. Determine the potential energy of the system with respect to (a) the spring, (b) gravity.



(a) SPRING

Length = AC + BC = $l \sin\theta + l \cos\theta$

Unstretched length = l

Elongation = $e = l\sin\theta + l\cos\theta - l = l(\sin\theta+\cos\theta - 1)$

$U_s = (1/2)\,ke^2 = (1/2)kl^2(\sin\theta+\cos\theta-1)^2$

(b) GRAVITY

$U_g = U_g(A) + U_g(B) = mg(-l\sin\theta) + mg(0)$

$U_g = -mg\,l\sin\theta$

DYNAMICS

PROBLEM N4/10

The force $F = (xi + yj)/(x^2 + y^2)$ acts on the particle $P(x,y)$ which moved in the (x,y) plane. (a) Prove that F is a conservative force, (b) derive the potential function $V(x.y)$ associated with F.

(a) $\bar{F} = \dfrac{x\bar{i} + y\bar{j}}{x^2 + y^2}$

$$F_x = \frac{x}{x^2 + y^2} \; ; \qquad F_y = \frac{y}{x^2 + y^2}$$

$$\frac{\partial F_x}{\partial y} = \frac{-2xy}{(x^2 + y^2)^2} \; ; \qquad \frac{\partial F_y}{\partial x} = \frac{-2xy}{(x^2 + y^2)^2}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \qquad \therefore \bar{F} \text{ is conservative}$$

(b) $F_x = \dfrac{-\partial V}{\partial x}$ $\qquad$ $F_y = \dfrac{-\partial V}{\partial y}$

$dV = -F_x dx$ $\qquad$ $dV = -F_y dy$

$V_x = -\displaystyle\int \frac{x}{x^2 + y^2}\, dx$ $\qquad$ $V_y = -\displaystyle\int F_y dy$

$V = -\frac{1}{2}\ln(x^2 + y^2) + f(y)$ $\qquad$ $V = -\frac{1}{2}\ln(x^2 + y^2) + g(x)$

$$f(y) = g(x) = \ln C$$

$$V = -\ln\left|C\sqrt{x^2 + y^2}\right|$$

$$V = \ln\left|\frac{C}{\sqrt{x^2 + y^2}}\right|$$

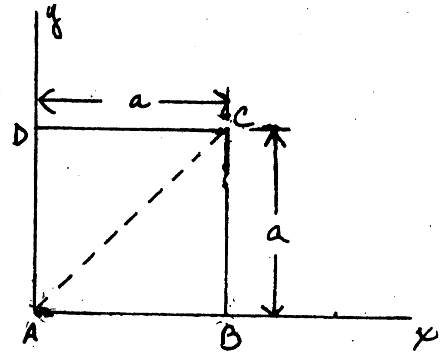
The force $F = x^2yi + xy^2j$ acts on the particle $P(x,y)$ which moves in the xy plane. Prove that F is a nonconservative force and determine the work of F as it moves from A to C along each of the paths ABC, ADC, and AC.

$\bar{F} = x^2y\bar{i} + xy^2\bar{j}$

$F_x = x^2y;\ F_y = xy^2$

$\dfrac{\delta F_x}{\delta y} = x^2;\quad \dfrac{\delta F_y}{\delta x} = y^2$

$\therefore\ \bar{F}$ is nonconservative



$$W = \int \bar{F}\cdot d\bar{r} = \int F_x dx + \int F_y dy = \int x^2y\,dx + \int xy^2\,dy$$

Path ABC

$${}_AW_B = \int_0^a x^2(0)dx = 0$$

$${}_BW_C = \int_0^a ay^2dy$$

$$= \frac{1}{3}a^4$$

$$\therefore W_{ABC} = \frac{1}{3}a^4$$

Path ADC

$${}_AW_D = \int_0^a (0)y^2dy = 0$$

$${}_DW_C = \int_0^a x^2(a)dx = \frac{1}{3}a^4$$

$$W_{ADC} = \frac{1}{3}a^4$$

Path AC

$x = y$

$${}_AW_C = \int x^2y\,dx + \int xy^2\,dy$$

$$= \int_0^a x^3dx + \int_0^a y^3dy$$

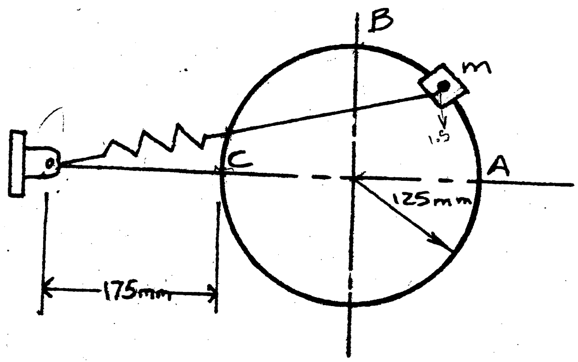
$$= \frac{a^4}{4} + \frac{a^4}{4} = \frac{a^4}{2}$$

$$W_{AC} = \tfrac{1}{2}a^4$$

DYNAMICS

PROBLEM N 4/12

A collar os mass 1.5 kg is attached to a spring and slides without friction along a circular rod which lies in a horizontal plane. The spring is undeformed when the collar is at C and the constant of the spring is 400 N/m. If the collar is released from rest at B, determine the velocity of the collar as it passes through point C.



$e_B = 150 = .15$ m

$e_C = 0$

$$E_B + \cancelto{0}{{}_BW_C} = E_C + \cancelto{0}{{}_BL_C}$$

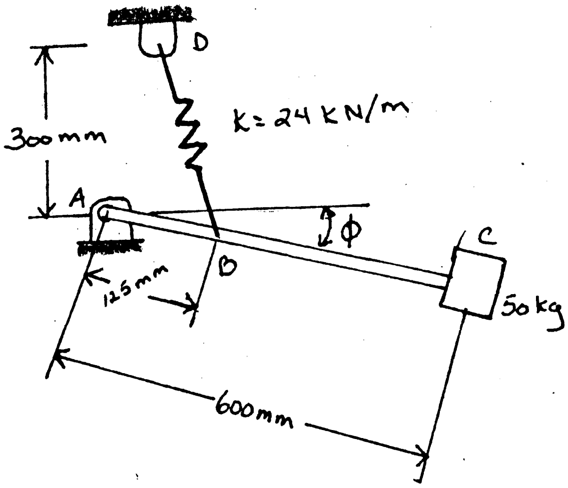
$$\tfrac{1}{2} k e_B^2 + \tfrac{1}{2} \cancelto{0}{m v_B^2} = \tfrac{1}{2} \cancelto{0}{k e_C^2} + \tfrac{1}{2} m v_C^2$$

$$\tfrac{1}{2} (400)(.15^2) = \tfrac{1}{2}(1.5) v_C^2$$

$$V_C^2 = 6$$

$$V_C = 2.45$$

A 50 kg block is released from rest when $\emptyset = 0$. If the speed of block when $\emptyset = 90^o$ is to be 2.5 m/s, determine the required value of the initial tension in the spring.



$E_1 = E_2$

position (1) $\emptyset = 0^0$

position (2) $\emptyset = 90^0$

$\frac{1}{2}ke_1^2 + \overset{0}{\cancel{\frac{1}{2}mv_1^2}} + mgh_1 = \frac{1}{2}ke_2^2 + \frac{1}{2}mv_2^2 + \overset{0}{\cancel{mgh_2}}$

$\frac{1}{2}ke_1^2 + 50(9.8)(.6) = \frac{1}{2}ke_2^2 + \frac{1}{2}(50)(2.5)^2$

$ke_1^2 + 588 = ke_2^2 + 312.5$

$e_2 = e_1 + .1$

$ke_1^2 + 588 = k(e_1 + .1)^2 + 312.5$

$ke_1^2 + 588 = ke_1^2 + .2ke_1 + .01k + 312.5$

$.2ke_1 + 0.1k = 275.5$

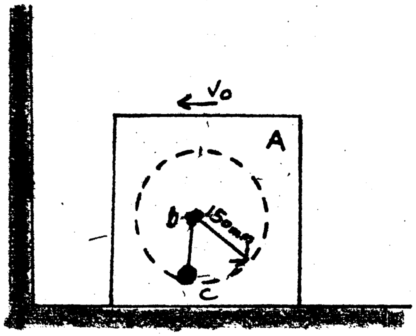
$ke_1 = 1377.5 - .05k$

$T = ke_1 = 1377.5 - .05(24\text{x}10^3)$

$T = 177.5$ N

The sphere C and the block A are both moving to the left with a velocity v_o when the block is suddenly stopped by the wall. Determine the smallest velocity v_o for which the sphere C will swing in a full circle about the pivot B (a) if BC is a slender rod of negligible weight, (b) if BC is a cord.

INITIAL VELOCITY OF C = V_o



(a) $E_1 = E_2$

Position (1)⟶directly below B

Position (2)⟶directly above B

V_c must be = 0

when C is directly above B

$\frac{1}{2}mv_o^2 = mgh$

$v_o^2 = 2gh$

$v_o^2 = 2(9.8)(.3)$

$v_o = 2.42$ m/sec

(b) For smallest v_o, the cord must become slack that is the tension becomes zero.

(c)

$\downarrow$ mg

$F_N = mg = ma_N$

$mg = m\,\dfrac{V_{c2}^2}{r}$

$v_{c2}^2 = gr$

$\frac{1}{2}mv_o^2 = \frac{1}{2}mv_{c2}^2 + mgh$

$v_o^2 = v_{c2}^2 + 2gh$

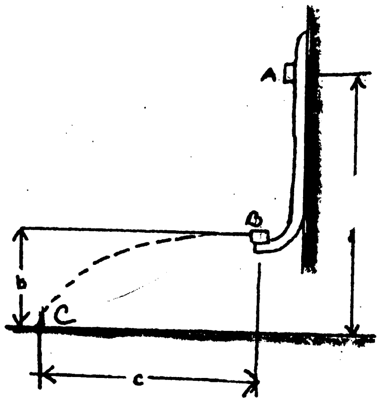
$V_o^2 = gr + 2gh$

$v_o^2 = 9.8\,(.15 + 2(.3))$

$v_o = 2.71$ m/sec

A small block is released at A with zero velocity and moves along the frictionless guide to point B where it leaves the guide with a horizontal velocity. Knowing that h = 3m and b = 1m, determine (a) the speed of the block as it strikes the ground at C, (b) the corresponding distance c.

$E_A = E_B$



(a) $mgh_A = mgh_B + \frac{1}{2}mv_B$

$g(3) = g(1) + \frac{1}{2}v_B^2$

$V_B^2 = 4g$

$V_B = 6.3$ m/s

$E_B = E_C$

$mgh_B + \frac{1}{2}mv_B^2 = \frac{1}{2}mv_C^2$

$v_C^2 = 2gh_B + V_B^2$

$V_C^2 = 2g + 4g$

$V_C = 6g$

$V_C^2 = 7.67$ m/sec

(b) $y = y_o + v_{oy}t - \frac{1}{2}gt^2$

$0 = 3 + 0 - \frac{1}{2}(9.8)(t^2)$

$t^2 = .61$

$t = .78$ sec

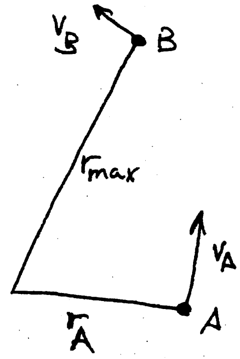
$x = x_o + v_o xt$

$v_{ox} = v_B = 6.3$

$x = 0 + (6.3)\ (.78)$

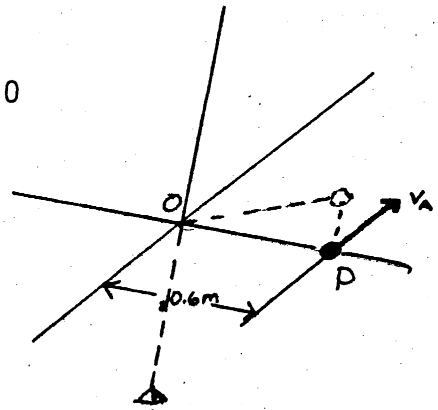
$x = 4.9$ m

A 2 kg sphere is attached to an elastic cord of constant 150N/m which is undeformed when the sphere is located at the origin O. Knowing that in the position shown, v_A is perpendicular to OP and has a magnitude of 10 m/s, determine (a) the maximum distance from the origin attained by the sphere, (b) the corresponding speed of the sphere.



At point B, $V_r = \frac{dr}{dt} = 0$

$\therefore V_B = V_\theta$



Conservation of Angular Momentum

$r_A(mV_A) = r_{max}\ (mV_B)$

$(0.6)(10m/s) = r_{max}(V_B)$

$V_B = \frac{6}{r_{max}}$

(a) $E_A = E_B$

$\frac{1}{2}\ mv_A^{\ 2} + \frac{1}{2}ke_A^{\ 2} = \frac{1}{2}\ mv_B^{\ 2} + \frac{1}{2}ke_B^{\ 2}$

$\frac{1}{2}(2)(10) + \frac{1}{2}(150)(.6^2) = \frac{1}{2}(2)(\frac{6}{r_{max}})^2 + \frac{1}{2}(150)(r_{max})^2$

solving for r_{max}

$r_{max} = 1.155$ m

(b) $V_B = \frac{6}{r_{max}}$

$V_B = 5.20$ m/sec