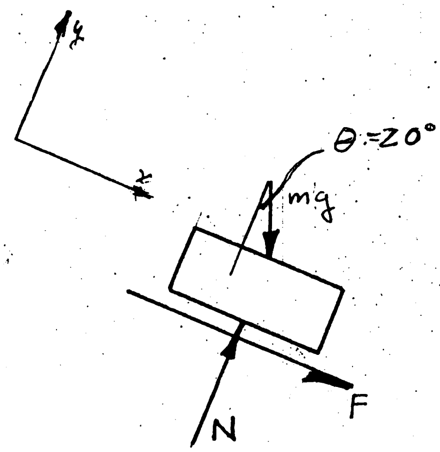
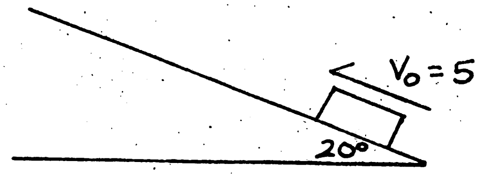
A 4 kg package is projected up a plane making a 20^0 angle with the horizontal. If the coefficient of friction is .3 and the package is projected with an initial velocity of 5 m/sec, how far up the plane will the package move before stopping?





$$\Sigma F_y = -mg \cos 20^0 + N = 0$$

$$N = mg \cos 20^0$$

$$\Sigma F_x = F + mg \sin 20^0 = ma$$

$$\mu \not{m}g \cos 20^0 + \not{m}g \sin 20^0 = \not{m}a$$

$$a = \mu g \cos 20^0 + g \sin 20^0$$

$$a = (.3)(9.8) \cos 20^0 + 9.8(\sin 20^0)$$

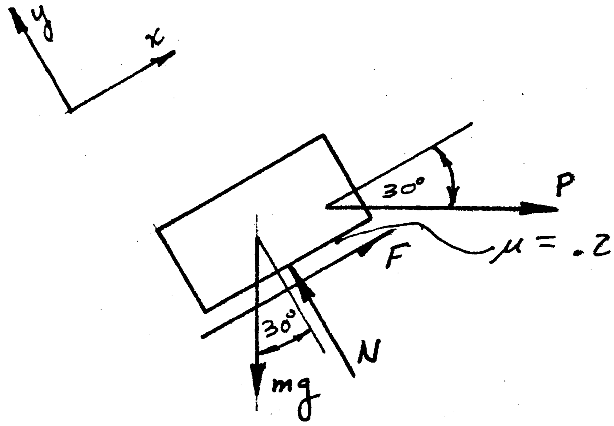
$$a = 6.1 \text{ m/sec}^2$$

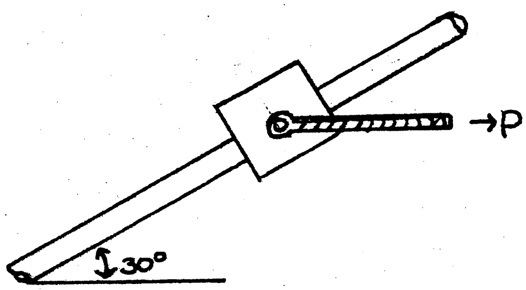
$$v_f^2 = v_o^2 + 2as$$

$$0 = 5^2 - 12.2(s)$$

$$s = 2 \text{ m}$$

A 3 kg collar moves down a rod with a velocity of 3 m/sec. A force P is applied to the horizontal cable. If μ = .20, between collar and rod find the magnitude of the force P so that the collar may be stopped after moving 1 m more down the rod.





$$v_f^2 = v_o^2 + 2a(X)$$

$$0 = 3^2 + 2a(-1)$$

$$a = +4.5$$

$$\Sigma F_y = N - mg\cos 30^\circ - P\sin 30^\circ = 0$$

$$N = mg\cos 30^\circ + P\sin 30^\circ$$

(1) $N = 25.5 + .5\,P$

$$\Sigma F_x = -mg\sin 30^\circ + \mu N + P\cos 30^\circ = ma$$

$$N = \frac{ma + mg\sin 30^\circ - P\cos 30^\circ}{\mu}$$

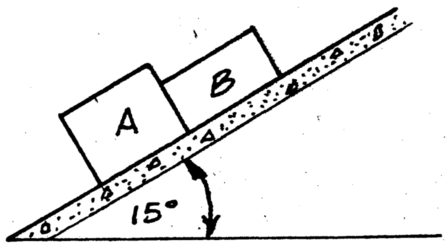
(2) $N = 67.5 + 73.5 - 4.3\,P$

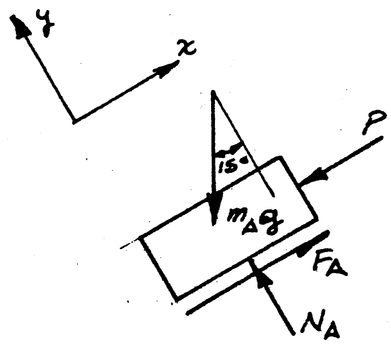
Combine equations (1) & (2)

$25.5 + .5P = 67.5 + 73.5 - 4.3P$

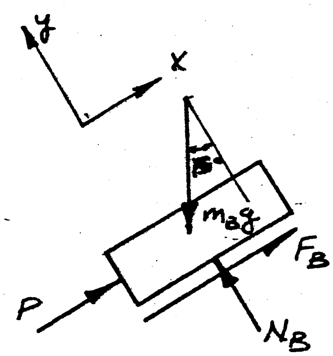
$P = 24$ N

Two crates A (10kg) and B (15kg) are placed on an inclined plane. The coefficient of friction between crate A and the incline is .2 while the coefficient of friction between crate B and the incline is .15. If the crates are initially in contact, find the acceleration of each crate and the contact force.





A:



B:

$\Sigma F_y = -98 \cos 15^\circ + N_A = 0$

$N_A = 94.7$

$\Sigma F_y = N_B - 147 \cos 15^\circ = 0$

$N_B = 142$

$\Sigma F_x = -98 \sin 15^\circ - P + .2(94.7) = 10a_A$

$10a_A = -P - 6.4$

$\Sigma F_x = .15(142) + P - 147 \sin 15^\circ = 15a_B$

$15a_B = p - 16.8$

Assume that the two blocks remain together.

$\therefore \quad a_B = a_A = a$

$10a = -P - 6.4$

$15a = P - 16.8$

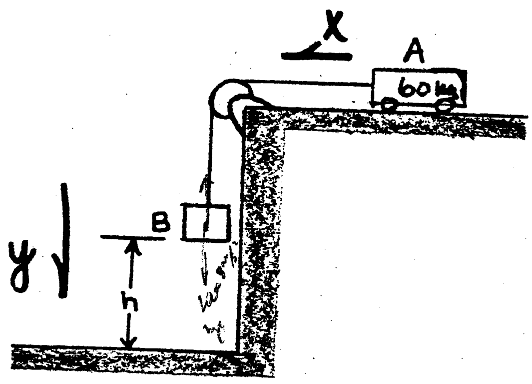
$25a = -23.2$

$a = -.93$

$P = 15a + 16.8$

$P = 2.9$ N

When the system shown is released from rest, the acceleration of block B is observed to be 3 m/s/s downward. Neglecting friction, find the tension in the cable and the mass of block B.



$\sum F_x = T = m_A a$

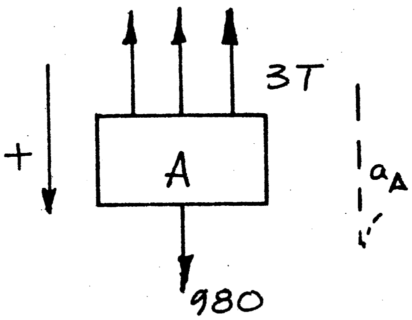
$T = 60(3) = 180$ N $\qquad T = 180$ N

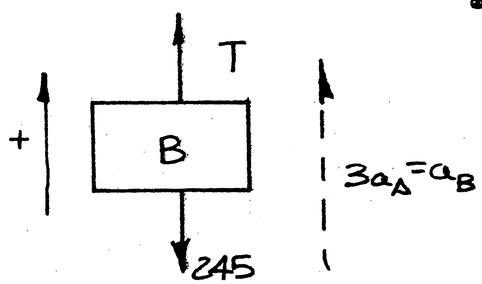
$\sum F_y = m_B g - T = m_B(3)$

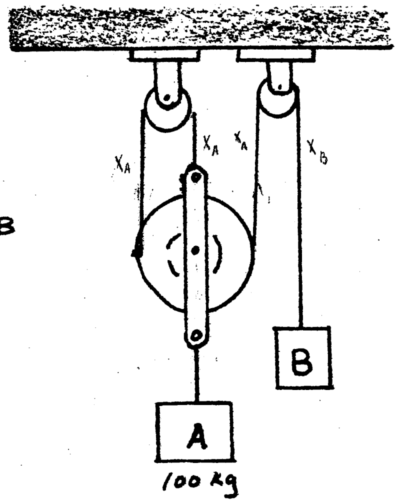
$m_B(9.8) - 180 = m_B(3)$

$m_B = 26.5$ kg

The 100 kg block A is connected to a 25 kg counterweight by the cable arrangement shown. If the system is released from rest, determine the tension in the cable, the velocity of B after 3 sec, and the velocity of A after it has moved 1.2 m.







A: $\sum F_y = 980 - 3T = 100a_A$

B: $\sum F_y = T - 245 = 25a_B$

$$980 - 3T = 100a_A$$
$$-245 + T = 75a_A$$

$a_A = .75 \downarrow \quad a_B = 2.26 \uparrow$

$$980 - 3T = 100a_A$$
$$-735 + 3T = 225a_A$$

$$245 = 325a_A$$

$$a_A = .75$$

$$T = \frac{225(.75) + 735}{3}$$

$$T = 301.5\ \text{N}$$

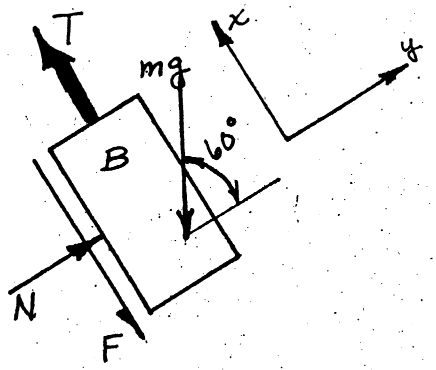
$$v_f = v_o + at$$
$$v_B = 0 + 2.26(3)$$

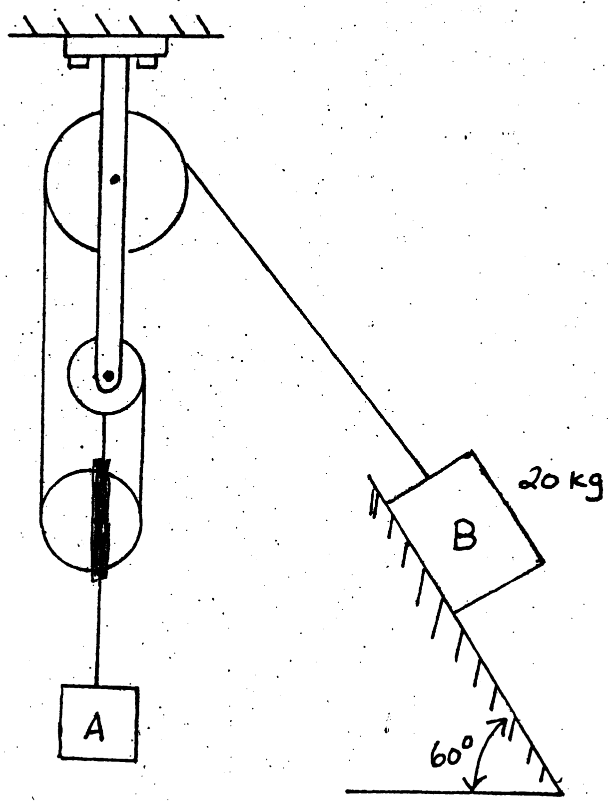
$$\boxed{v_B = 6.78\ \text{m/sec}}$$

$$v_f^2 = v_o^2 + 2as$$
$$v_f^2 = 2(.75)(1.2)$$

$$\boxed{v_f = 1.3\ \text{m/sec}}$$

Determine the weight of block A, required to move block B 3m up the inclined plane in 4 sec. starting from rest. Neglect the weights of the pulleys and cable.



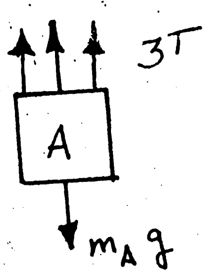


B:

$$F_x = T - 196 \sin 60^0 = ma$$

$$T - 196 \sin 60^0 = 20a_B$$

$$T = 177.2 \text{ N}$$



A: $a_A = \frac{1}{3} a_B$

$$a_A = .125 \text{ m/sec}^2$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$3 = 0 + \tfrac{1}{2}a(4)^2$$

$$a_B = .375 \text{ m/sec}^2$$

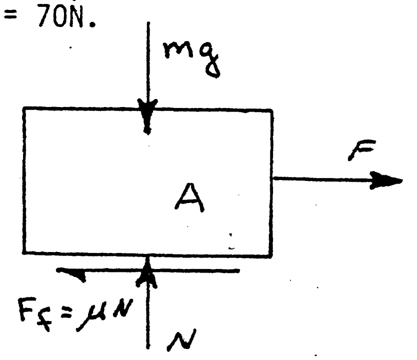
$$F_y = 3T - mg = ma_A$$

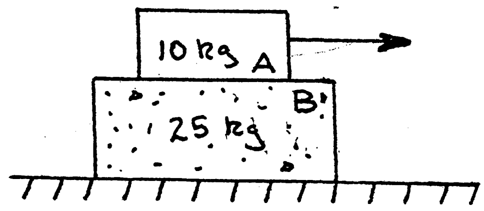
$$3T = mg + ma_A$$

$$m = \frac{3T}{g+a}$$

$$m = 53.56 \text{ kg}$$

Block B rests upon a smooth surface. If the coefficient of friction between A and B is $\mu = 0.4$, determine the acceleration of each block if (a) F = 35N and (b) F = 70N.





(a) BODY A

$\Sigma F_y = N - mg = 0$

$N = 98$

$\Sigma F_x = F - \mu N = ma$

ASSUME a = 0
$F_f = 39.2$
$F_{APPLIED} < F_f$
A WON'T SLIDE

$F_f \leq 39.2$ F = 35

∴ Block 'A' will not slide across block 'B'. Both blocks will act as one mass

$\Sigma F_x = 35 = ma$

$35 = 35a$

$a = 1 m/sec^2$

(b) BODY A

$\Sigma F_x = F - \mu N = m_A a_A$

$10_a = 70 - 39.2 = 30.8$

$a_A = 3.08 m/sec^2$

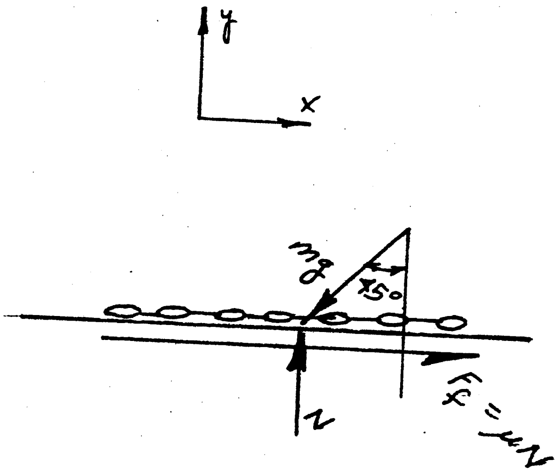
BODY B

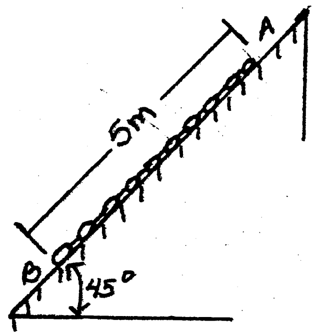
$\Sigma F_x = \mu N = m_B a_B$

$39.2 = 25 a_B$

$a_B = 1.57 m/sec^2$

The chain is 5m long and weighs 3kg/m. If $\mu = 0.22$, determine the velocity at which the end A will pass point B when the chain is released from rest.





$m = 3\ \text{kg/m} \times 5\ \text{m}$

$m = 15\ \text{kg}$

$\Sigma F_y = N - mg\cos 45^\circ = 0$

$N = mg\cos 45^\circ$

$N = 103.9 \qquad \mu N = 22.9$

$\Sigma F_x = 22.9 - mg\sin 45^\circ = 15\,a$

$$a = \frac{22.9 - mg\sin 45^\circ}{15}$$

$a = -5.4$

$a = 5.4\ \text{m/sec}^2$ down slope

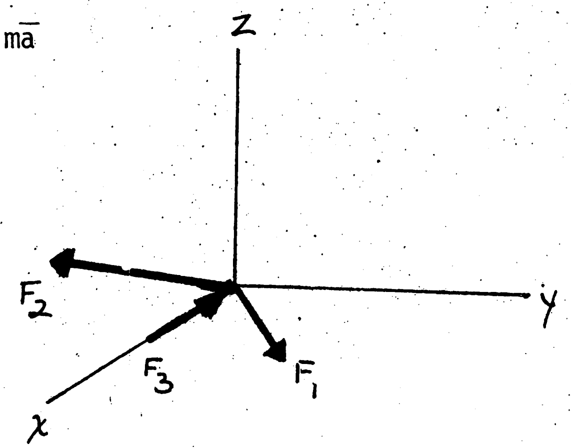
$v_f^2 = \overset{0}{\cancel{v_0^2}} + 2\,as$

$v_f = \sqrt{2(5.4)(5)}$

$v_f = 7.35\ \text{m/sec}$

A 3 kg ball is subjected to the action of forces: $F_1 = 2i \pm 6j - 2tk$, $F_2 = t^2 i - 4tj - k$, and $F_3 = -2ti$, where the force is given in newtons and time in seconds. Determine the magnitude of displacement of the ball with respect to its original position 2 sec. after being released from rest. Assume that the directions of the forces acting on the ball are constant during the time interval.

$\Sigma \bar{F} = m\bar{a}$



$$\Sigma \bar{F} = (t^2 - 2t + 2)\bar{i} + (6 - 4t)\bar{j} + (-1-2t)\bar{k} = m\bar{a}$$

$$(t^2 - 2t + 2)\bar{i} + (6 - 4t)\bar{j} - (2t + 1)\bar{k} = 3\left(\frac{dv_x}{dt}\bar{i} + \frac{dv_y}{dt}\bar{j} + \frac{dv_z}{dz}\bar{k}\right)$$

$3\frac{dv_x}{dt} = t^2 - 2t + 2$ by integration $3\frac{ds_x}{dt} = \frac{t^3}{3} - t^2 + 2t + Cx$

$3\frac{dv_y}{dt} = 6 - 4t$ $\qquad 3\frac{ds_y}{dt} = 6t - 2t^2 + Cy$

$3\frac{dv_z}{dt} = -(2t+1)$ $\qquad 3\frac{ds_z}{dt} = -(t^2 + t) + Cz$

At t = 0, the components of velocity equal zero

$\therefore$ Cx, Cy, and Cz = 0

$$3sx = \frac{t^4}{12} - \frac{t^3}{3} + t^2 + Cx'$$

$$3sy = 3t^2 - \frac{2t^3}{3} + Cy'$$

$$3sz = -\left(\frac{t^3}{3} + \frac{t^2}{2}\right) + Cz'$$

At t = 0; sx = sy = sz = 0

$\therefore$ cx' = cy' = cz' = 0

At t = 2

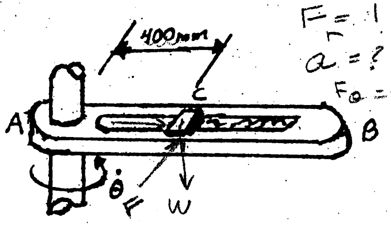
Sx = .22; Sy = 2.22; Sz = 1.56

$S = \sqrt{sx^2 + sy^2 + sz^2}$

S = 2.72 m

Slider C has a mass of 250 g and oscillates in the radial slot in arm AB as the arm rotates in a horizontal plane at a constant rate of $d\theta/dt = 12$ rad/s. In the position shown, it is known that the slider is moving outward along the slot at the speed of 1.5 m/s and that the spring is compressed and exerts a force of 10 N on the slider. Neglecting the effect of friction, determine (a) the components of the acceleration of the slider, (b) the horizontal force exerted on the slider by the arm AB.

$\dot\theta = 12$ $\ddot\theta = 0$
$\dot r = 1.5$
$F_r = 10$
$a = ?$
$F_\theta = ?$



(a) $F_r = m\,a_r$

with $F_r = -10$ N, $m = .250$ kg

$-10 = .250\, a_r$

$a_r = -40\ \text{m/s}^2$

On the other hand

$$a_\theta = r\ddot\theta + 2\,\dot r\,\dot\theta$$

with

$r = .4$ m, $\dot r = 1.5$ m/s $\dot\theta = 12$ rad/sec = const. $\ddot\theta = 0$

$a_\theta = (.4)(0) + 2(1.5)(12)$

$a_\theta = +36\ \text{m/s}^2$

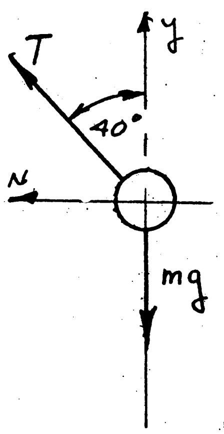
Note that $\ddot r \neq 0$. Indeed, we have

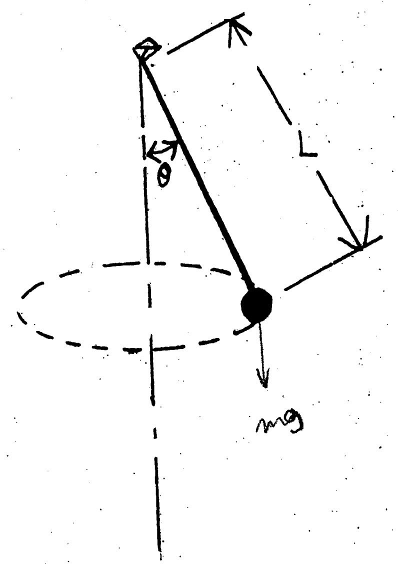
$a_r = \ddot r - r\dot\theta^2$ $\ddot r = a_r + r\dot\theta^2 = 40 + (.4)(12)^2 = 17.6\ \text{m/s}^2$

(b) $F_\theta = ma_\theta = (.25)(36)$

$F_\theta = +9$ N.

The small ball of mass 5 kg is attached to a cord of length 2m and is made to revolve in a horizontal circle at a constant speed v_o. Knowing that the cord forms an angle $\theta = 40^o$ with the vertical, determine the tension in the cord, and the speed v_o of the ball.





$$\sum Fy = T \cos 40^o - 5\,(9.8) = 0$$

$$T = \frac{5(9.8)}{\cos 40^o}$$

$$T = 63.9N$$

$$\sum F_N = T \sin 40 = ma$$

$$(63.9)(\sin 40^o) = 5\left(\frac{v_o^2}{\rho}\right)$$

$$\rho = L \sin 40^o$$

$$\rho = 2 \sin 40^o$$

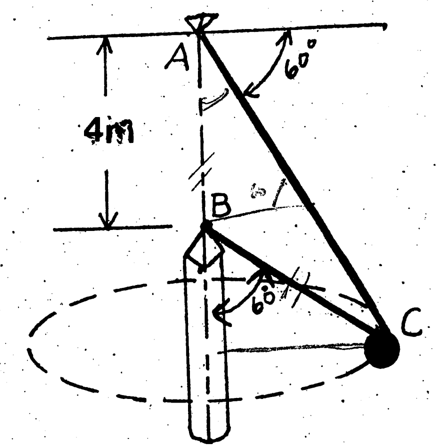
$$41.1 = 5\left[\frac{v_o^2}{2(\sin 40^o)}\right]$$

$$v_o = 3.3 \text{ m/s}$$

12.43

DYNAMICS PROBLEM N3/12

Two wires AC and BC are each tied to a sphere at C. The sphere is made to revolve in a horizontal circle at a constant speed v. Determine the range of values of the speed v for which both wires are taut.



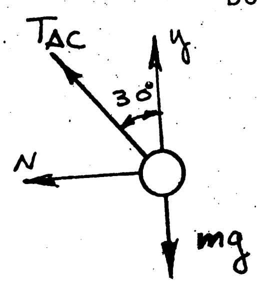
ABC is isosceles

$\therefore$ AB = BC = 4 m

$\rho = 4 \sin 60^{\circ}$

$\rho = 3.46$

Find V for $T_{BC} = 0$



$\sum F_y = T_{AC} \cos 30^{\circ} - mg = 0$

$T_{AC} = \dfrac{mg}{\cos 30^{\circ}}$

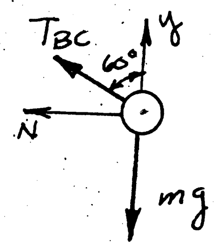
$\sum F_N = T_{AC} \sin 30 = ma$

$\dfrac{mg \sin 30}{\cos 30} = m \dfrac{v^2}{\rho}$

$V = \sqrt{\rho g \tan 30^{\circ}}$

V = 4.42 m/s

Find V for $T_{AC} = 0$



$\sum F_y = T_{BC} \cos 60 - mg = 0$

$T_{BC} = \dfrac{mg}{\cos 60^{\circ}}$

$\sum F_N = T_{BC} \sin 60 = m \dfrac{v^2}{\rho}$

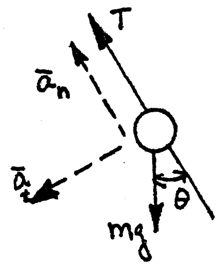
$\dfrac{mg \sin 60}{\cos 60} = m \dfrac{v^2}{\rho}$

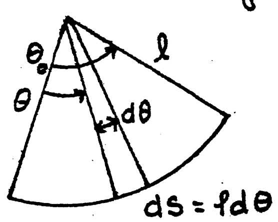
$V \sqrt{\rho g \tan 60^{\circ}}$

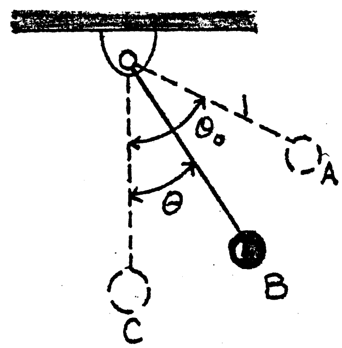
V = 7.66 m/s

$4.47 \leq V \geq 7.66$

A ball (weight W) is released from rest from positon A and oscillates in a vertical plane at the end of a cord of length 1. Find: (a) the tangential component of the acceleration at B in terms of θ, (b) the velocity at B in terms of θ, θ_0 and ℓ, (c) the tension θ in $_0$ and at C as a function of W and θ_0, (d) the value of θ_0 if T = 2W at C.







(a) $\Sigma F_t = mg\sin\theta = ma_t$

$a_t = g\sin\theta$

(b) $a_t = V\frac{dv}{ds}$

$vdv = a_t ds \qquad ds = -\ell d\theta$

$$\int_0^V vdv = -\int_{\theta_0}^{\theta} \ell g \sin\theta d\theta$$

$$\frac{V^2}{2} = g\ell\cos\theta \Big|_{\theta_0}^{\theta}$$

$$\frac{1}{2}V^2 = g\ell(\cos\theta - \cos\theta_0)$$

$$V = \sqrt{2g\ell(\cos\theta - \cos\theta_0)}$$

(c) $\theta = 0 \qquad a_t = g\sin 0^0 = 0$

$a_n = \frac{V^2}{\rho}$

$$\Sigma F_n = T - W = m\frac{V^2}{\ell}$$

$$V^2 = 2g\ell(\cos 0^0 - \cos\theta_0)$$

$$T - W = \frac{W}{g}\frac{2g\ell(1 - \cos\theta_0)}{\ell}$$

$$T - W = 2W(1 - \cos\theta_0)$$

$$T = W(3 - 2\cos\theta_0)$$

(d) IF T = 2W

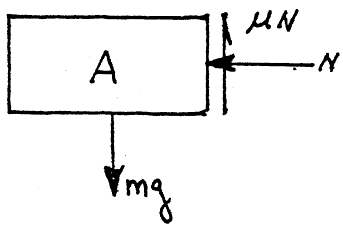
$$2W = W(3 - 2\cos\theta_0)$$

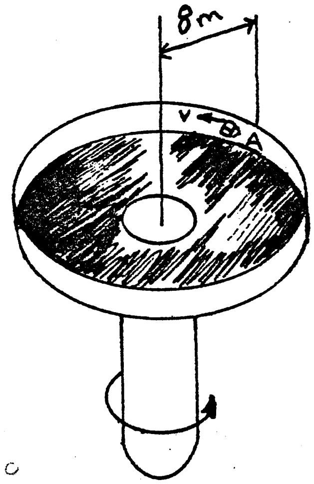
$$-W = -2W\cos\theta_0$$

$$\cos\theta_0 = \frac{1}{2}$$

$$\theta_0 = 60^0$$

The assembly shown rotates about a vertical axis at a constant rate. Knowing that the coefficient of friction between the small block A and the cylindrical wall is 0.20, determine the lowest speed v for which the block will remain in contact with the wall.





$\ddot{\phi} = 0$

$$\Sigma F_N = ma_N = N$$

$$\Sigma F_y = -mg + \mu N = 0$$

$$.2ma_N = mg$$

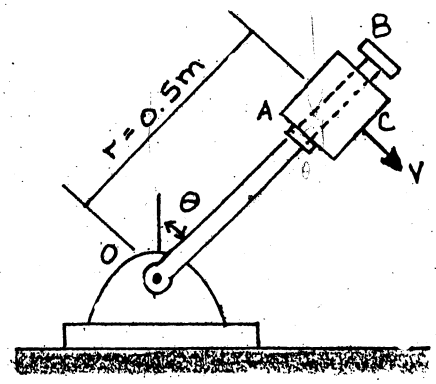
$$a_N = \frac{9.8}{.2} \qquad a_n = \frac{v^2}{\rho}$$

$$a_N = 49 m/s^2$$

$$V = \sqrt{a_N \rho} = \sqrt{49(8)}$$

$$\underline{\underline{V = 19.8 m/s}}$$

Rod OAB rotates in a vertical plane at a constant rate so that the speed of collar C is 1.5m/s. The collar is free to slide on the rod between two stops A and B. Knowing that the distance between the stops is only slightly larger than the collar and neglecting the effect of friction, determine the range of value of θ for which the collar is in contact with stop A.



$$\Sigma F_n = ma_n$$

$$mg\cos\theta - F_A = ma_n$$

$$F_A = m(g\cos\theta - 4.5)$$

$$a_n = \frac{v^2}{\rho} = \frac{(1.5)^2}{.5} = 4.5 m/s^2$$

For Contact $F_A > 0$

or $g\cos\theta > 4.5$

$$\cos\theta > \frac{4.5}{9.81}$$

$$-62.7^0 < \theta \leq 62.7^0$$

The motion of a 2 kg particle is defined by the equation $r = t^3 - 2t^2$ and $\theta = 3t^3$. Find the force on the particle when t = 2s.

$$\theta = 3t^3$$

$$r = t^3 - 2t^2$$

$$\frac{d\theta}{dt} = 9t^2 \qquad \frac{dr}{dt} = 3t^2 - 4t$$

$$\frac{d^2\theta}{dt^2} = 18t \qquad \frac{d^2r}{dt^2} = 6t - 4$$

$$\bar{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\bar{u}_r + \left[2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}\right]\bar{u}_\theta$$

$$\bar{a} = \left[(6t-4) - (t^3-2t^2)(9t^2)^2\right]\bar{u}_r + \left[2(3t^2-4t)(9t^2)+(t^3-2t^2)(18t)\right]\bar{u}_\theta$$

AT t = 2 sec

$$\bar{a} = 8\,\bar{u}_r + 288\,\bar{u}_\theta$$

$$\bar{F} = m\bar{a}$$

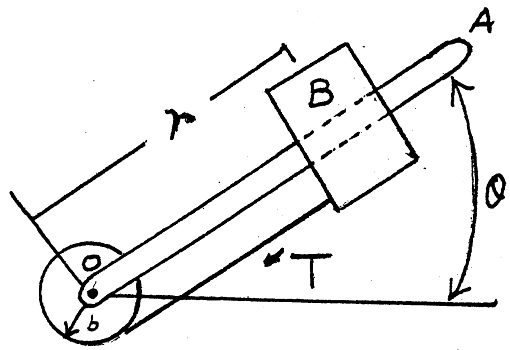
$$m = 2 \text{ kg}$$

$$\bar{F} = 16\,\bar{u}_r + 576\,\bar{u}_\theta$$

$$F = \sqrt{16^2 + 576^2} = 576.2\text{N}$$

X

A block B of mass m may slide on the frictionless arm OA which rotates in a horizontal plane at a constant velocity dθ/dt. The drum around which the cord wraps is fixed. Express as a function of M, r, b and dθ/dt, (a) the tension T in the cord, (b) the magnitude of the horizontal force Q exerted on B by the arm OA.



$$r = r_o - b\dot{\theta}t$$

$$\dot{r} = -b\,\dot{\theta}$$

$$\ddot{r} = 0$$

$$\dot{\theta} = \text{constant}$$

$$\ddot{\theta} = 0$$

$$\Sigma F_r = m\,a_r$$

$$-T = m(\ddot{r} - r\dot{\theta}^2) = -m(0 - r\dot{\theta}^2)$$

$$T = mr\dot{\theta}^2$$

$$\Sigma F_\theta = m\,a_\theta$$

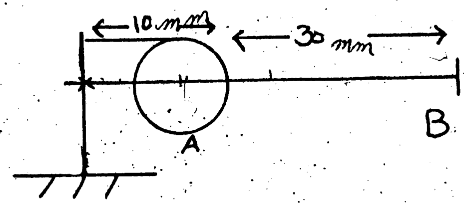
$$Q = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = m(0 - 2b\dot{\theta}^2)$$

$$Q = -2mb\dot{\theta}^2$$

$$|Q| = 2mb\dot{\theta}^2$$

A heavy ball is mounted on a horizontal rod which rotates freely about a vertical shaft. The ball has an initial speed $V_a = 1$ m/s while held in position by a cord. If the cord is cut, what is the speed of the ball when it reaches point B. Write the equation of the path from A to B.

Conservation of ang. momentum



$H_o = 10\ m\mathbf{v}_1 = 40\ m\mathbf{v}_2$

$V_2 = \frac{1}{4} V_1$

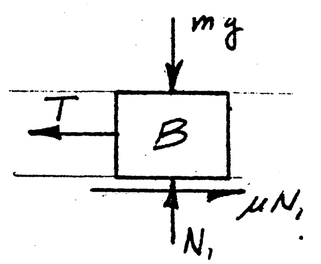
$= \frac{1}{4}$ m/s

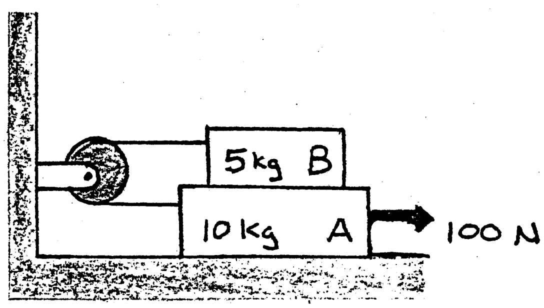
since no force acts on the ball until it hits stop B.

$X = X_o + vt$

$= .01 + \frac{1}{4}\ t$

Knowing that the coefficient of friction is 0.30 at all surfaces of contact, determine (a) the acceleration of plate A, (b) the tension in the cable. (Neglect bearing friction in the pulley.)



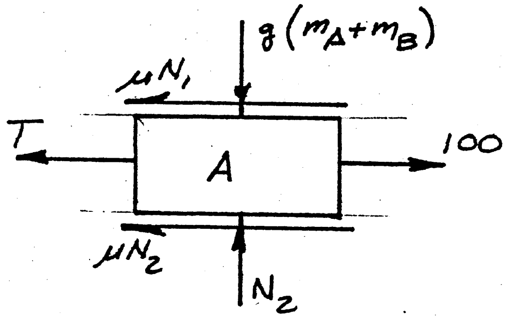


B: $\Sigma F_y = N_1 - mg = 0$

$N_1 = m_B g$

$\Sigma F_x = T - \mu N_1 = m_B a_B$

$T = \mu m_B g + m_B a_B$



A: $\Sigma F_y = N_2 - (m_A + m_B) g = 0$

$N_2 = (m_A + m_B) g$

$\Sigma F_x = 100 - T - \mu N_1 - \mu N_2 = m_A a_A$

$T = 100 - \mu m_B g - \mu (m_A + m_B) g - m_A a_A$

$a_A = a_B$ (Note coordinate system)

(a) $T = \mu m_B g + m_B a_A$

$T = 100 - \mu m_B g - \mu (m_A + m_B) g - m_A a_A$

$\therefore$

$\mu m_B g + m_B a_A = 100 - \mu m_B g - \mu (m_A + m_B) g - m_A$

$a_A = \dfrac{100 - \mu m_B g - \mu g (m_A + m_B) - \mu m_B}{(m_A + m_B)}$

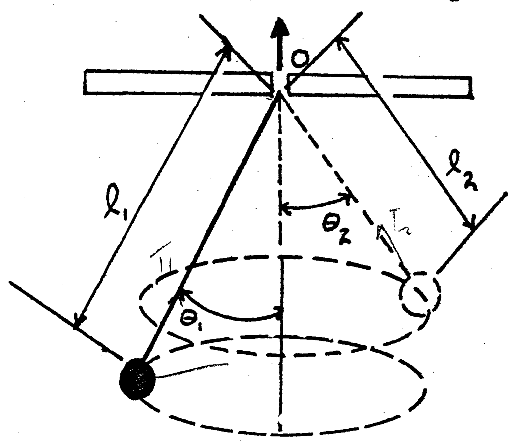
SUBSTITUTING VALUES

$a_A = 1.77$ m/sec

(b) $T = \mu m_B g + m_B a_A$

$T = 23.5$

A small ball swings in a horizontal circle at the end of a cord of length ℓ_1, which forms an angle θ_1 with the vertical. The cord is then slowly drawn through the support at O until the free end is ℓ_2. Derive a relation between ℓ_1, ℓ_2, θ_1 and θ_2.



$$\dot{H}_z = 0$$

$$R_1 m v_1 = R_2 m v_2$$

$$(\ell_1 \sin\theta_1) V_1 = (\ell_2 \sin\theta_2) V_2$$

$$(\ell_1 \sin\theta_1)^2 V_1^2 = (\ell_2 \sin\theta_2)^2 V_2^2$$

$$\Sigma F_n = m a_n$$

$$T_1 \sin\theta_1 = \frac{m v_1^2}{R_1}$$

$$V_1^2 = \frac{T_1 \ell_1 \sin^2\theta_1}{m}$$

$$g \ell_1 \sin\theta_1 \tan\theta_1 (\ell_1 \sin\theta_1)^2 =$$

$$g \ell_2 \sin\theta_2 \tan\theta_2 (\ell_2 \sin\theta_2)^2$$

$$\Sigma F_z = 0$$

$$T_1 \cos\theta_1 = mg$$

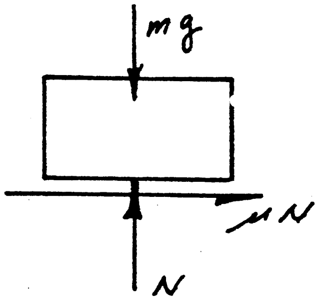
$$T_1 = \frac{mg}{\cos\theta_1}$$

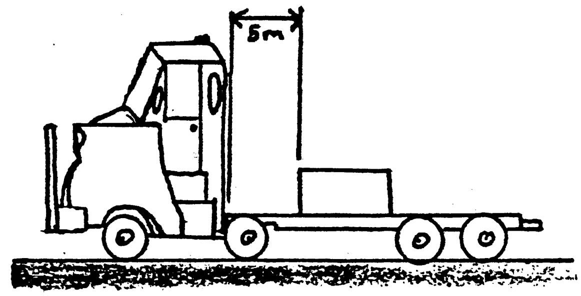
$$\ell_1^3 \sin^3\theta_1 \tan\theta_1 = \ell_2^3 \sin^3\theta_2 \tan\theta_2$$

$$\therefore V_1^2 = g \ell_1 \sin\theta_1 \tan\theta_1$$

Similarly $V_2^2 = g \ell_2 \sin\theta_2 \tan\theta_2$

The coefficient of friction between the load and the flat-bed trailer shown is 0.40. Knowing that the forward speed of the truck is 50 km/h, determine the shortest distance in which the truck can be brought to a stop if the load is not to shift.





$\Sigma F_y = N - mg = 0$

$N = mg$

$\Sigma F_x = \mu N = ma$

$a = \frac{\mu N}{m}$

$a = \frac{mg\mu}{m}$

$a = g\mu$

$a = 3.92\ m/sec^2$

$v = 50\ km/hr = 13.89\ m/sec$

$v^2 = v_0^2 + 2as$

$0 = (13.89)^2 + 2(-3.92)(s)$

$s = 24.6\ m$

THE VALUE OF (S) CORRESPONDIN TO THE LARGEST PERMISSIBLE (a) IS SMALLEST PERMISSIBLE DISTANCE IN WHICH THE TRUCK MAY STOP.