PROBLEM N2/1

If the displacement of a particle is given by $\bar{s} = (3t^3 - 2t)\bar{i} + (t^2 - 5)\bar{j}$, find the velocity and acceleration when t = 2 sec.

$$\bar{S} = (3t^3 - 2t)\bar{i} + (t^2 - 5)\bar{j}$$

$$\frac{d\bar{s}}{dt} = \bar{v} = (9t^2 - 2)\bar{i} + (2t)\bar{j}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = (18t)\bar{i} + 2\bar{j}$$

At t = 2 sec

$$\bar{v} = 34\bar{i} + 4\bar{j}$$

$$\bar{a} = 36\bar{i} + 2\bar{j}$$

If the displacement of a particle is given by $x = t^3 - e^{-t}$ and $y = t/(1+t)$, what is the velocity and acceleration at t=2?

$x = t^3 - e^{-t}$

$v_x = dx/dt = 3t^2 + e^{-t}$

$a_x = dv_x/dt = 6t - e^{-t}$

$y = t/(1+t)$

$v_y = dy/dt = \dfrac{(t+1)-t}{(t+1)^2} = \dfrac{1}{(t+1)^2}$

$a_y = dv_y/dt = -2(t+1)^{-3} = \dfrac{-2}{(t+1)^3}$

@t=2, $v_x = 12.14$ $v_y = 0.11$

$a_x = 11.86$ $a_y = -0.07$

$\bar{V} = v_x\bar{i} + v_y\bar{j}$

$12.14\bar{i} + 0.11\bar{j}$

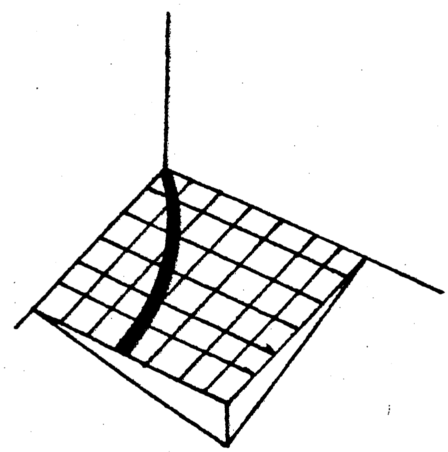
$\bar{a} = a_x\bar{i} + a_y\bar{j}$

$= 11.86\bar{i} - 0.07\bar{j}$

DYNAMICS

PROBLEM N2/3

The three-dimensional motion of a particle is defined by the positon vector $\bar{r} = At\bar{i} + ABt^3\bar{j} + Bt^2\bar{k}$, where r is expressed in meters, and t in seconds. Show that the space curve described by the particle lies on the hyperbolic paraboloid $y = xz$. For A = B = 1, determine the magnitudes of the velocity and acceleration when (a) t = 1sec, (b) t = 2sec.



$$\bar{r} = At\bar{i} + ABt^3\bar{j} + Bt^2\bar{k}$$

$$x = At \qquad y = ABt^3 \qquad z = Bt^2$$

$$xz = (At)(Bt^2) = ABt^3 = y$$

For A = B = 1

$$\bar{r} = t\bar{i} + t^3\bar{j} + t^2\bar{k}$$

$$\bar{v} = \bar{i} + 3t^2\bar{j} + 2t\bar{k}$$

$$\bar{a} = 6t\bar{j} + 2\bar{k}$$

$$V = \sqrt{1 + (3t^2)^2 + (2t)^2}$$

$$a = \sqrt{(6t)^2 + (2)^2}$$

t = 1 sec

$$V = \sqrt{1 + 9 + 4} = 3.74 \text{ m/sec}$$

$$a = \sqrt{36 + 4} = 6.32 \text{ m/sec}^2$$

t = 2 sec

$$V = \sqrt{1 + 144 + 16} = 12.69 \text{ m/sec}$$

$$a = \sqrt{144 + 4} = 12.17 \text{ m/sec}^2$$

PROBLEM N2/4

The displacement of a particle is given by $\bar{r} = (R \sin pt)\bar{i} + Rt\bar{k}$. Find the velocity and acceleration of the particle.

$$\bar{r} = (R \sin pt)\bar{i} + Rt\bar{k}$$

$$\bar{V} = \frac{d\bar{r}}{dt} = (p\ R \cos pt)\bar{i} + R\bar{k}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = (-p^2\ R \sin pt)\bar{i}$$

PROBLEM N2/5

A man standing on a bridge 20m above the water throws a stone in a horizontal direction. Knowing that the stone hits the water 30m from a point on the water directly below the man, determine (a) the initial velocity of the stone, (b) the distance at which the stone would hit the water if it were thrown with the same velocity from a bridge 5m lower.

a) VERTICAL MOTION

$y_o = 0 \quad V_{yo} = 0$

$y = \frac{1}{2} gt^2$

$20 = (\frac{1}{2})(9.81)t^2$

$t = 2.019$ sec

HORIZONTAL MOTION

$x = (V_o)_x t$

$V_o = \frac{30}{2.019} = 14.86$ m/sec

b) $y = \frac{1}{2} gt^2$

$t = \sqrt{\frac{(2)(15)}{9.81}}$

$t = 1.749$ sec

$x = V_o t$

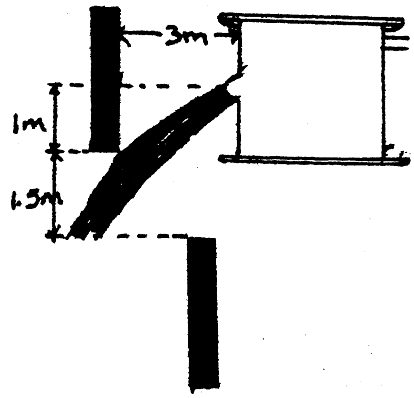
$x = (14.86)(1.749)$

$x = 25.99$m

DYNAMICS

PROBLEM N2/6

Water issues at A from a pressure tank with a horizontal velocity v_o. For what range of values v_o will the water enter the opening BC?



$1_m \le y < 2.5_m$

$$y = \frac{1}{2} g t^2$$

$$\sqrt{\frac{(1)(2)}{9.81}} = t_1$$

$$\sqrt{\frac{(2.5)(2)}{9.81}} = t_2$$

$.452_{sec} \le t < .714_{sec}$

$$x = V_o t$$

$$V_{o1} = \frac{3}{.452}$$

$$V_{o2} = \frac{3}{.714}$$

$4.20_{m/sec} < V_o \le 6.64_{m/sec}$

DYNAMICS

PROBLEM N2/7

A projectile is fired so that it lands 3000m on level ground. What is the minimum speed at which the projectile can be fired to achieve this range?

$$R = \frac{V_o^2 \sin 2\theta}{g}$$ ✓

$$V_o^2 = \frac{Rg}{\sin 2\theta}$$

$$V_o = (Rg \csc 2\theta)^{1/2}$$

$$\frac{dV_o}{d \csc 2\theta} = \frac{1}{2} (Rg \csc 2\theta)^{-1/2} (-\csc 2\theta \cot 2\theta) = 0$$

$$\frac{1}{\sqrt{Rg \csc 2\theta}} = 0$$

$$\frac{1}{Rg \csc 2\theta} = 0$$

$$\sin 2\theta = 0$$

$$\theta = 45^0$$

$$V_o = \sqrt{\frac{Rg}{\sin (2.45)}}$$ $\overset{?}{=} \sin 90^0$

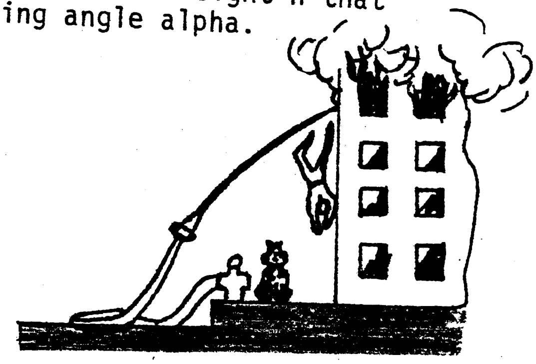
$$V_o = \sqrt{29400}$$

$$\underline{\underline{V_{o\ min} = 171.5}}\ \text{m/sec}$$

DYNAMICS

PROBLEM N2/8

A nozzle discharges water with an initial velocity of 20m/s. Knowing that the nozzle is 30m from a building, find (a) the maximum height h that can be reached by the water and (b) the corresponding angle alpha.



$V_{xo} = 20 \cos\alpha$ $V_{yo} = 20 \sin\alpha$

$x = V_{xo} t$

$t = \frac{3}{2 \cos\alpha}$

$h = V_{yo} t - \frac{1}{2} g t^2$

$= 30 \tan\alpha - 11.04 \sec^2\alpha$

$= 30 \tan\alpha - 11.04 \tan^2\alpha - 11.04$

h is max when $\frac{dh}{d(\tan\alpha)} = 0$

$\frac{dh}{d(\tan\alpha)} = 30 - 22.08 \tan\alpha = 0$

$\tan\alpha = \frac{30}{22.08}$

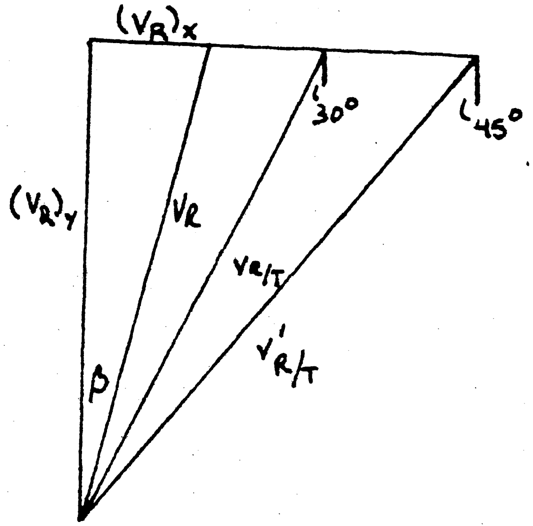
$\alpha = 53.65^0$

$h = 30\left(\frac{30}{22.08}\right) - (11.04)\left(\frac{30}{22.08}\right)^2 - 11.04$

$h = 9.3m$

PROBLEM N2/9

During a rainstorm the paths of the raindrops appear to form an angle of 30° with the vertical when observed from a side window of a train moving at a speed of 15km/h. A short time later, after the speed of the train has increased to 30km/h, the angle between the vertical and the paths of the drops appear to be 45°. If the train were stopped, at what angle and with what velocity would the drops be observed to fall?



For $V_{train} = 15 km/h$

$V_R = V_t' + V_{R/t}'$

For $V_t = 30$ $\quad V_R = V_t' + V_{R/T}'$

(1) $(V_R)_y \tan 45^o = (V_R)_x + 30$

(2) $(V_R)_y \tan 30^o = (V_R)_x + 15$

SUBTRACTING

$(V_R)_y (\tan 45^o - \tan 30^o) = 15$

$(V_R)_y = \frac{15}{.422} = 35.49$ km/h

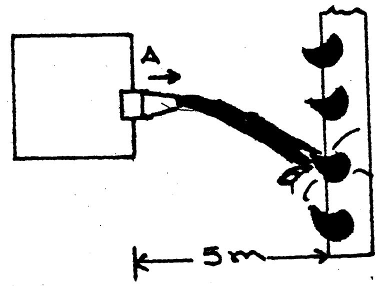
From (1)

$(V_R)_x = 35.49 - 30 = 5.49$ km/h

$\beta = \tan^{-1} \frac{5.49}{35.49} = 8.8^o$

$V_R = \frac{35.49}{\cos B} = 35.91$ km/h

Water is discharged at a with an initial velocity of 10m/s and strikes a series of vanes at B. Knowing that the vanes move downward with a constant speed of 3 m/s, determine the velocity and acceleration of the water relative to the vane at B.



HORIZONTAL MOTION

$V_x = 10$ m/s

$x = V_x t$

at B $\quad 5 = 10t$

$t = \frac{1}{2}$ sec

VERTICAL MOTION

$V_y = gt = \frac{9.81}{2} = 4.91\text{m/s}\downarrow$

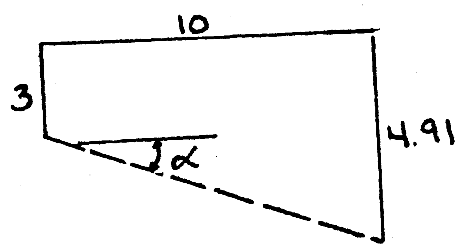
at B $V_w = 10\text{m/s}\rightarrow + 4.91\text{m/s}\downarrow$

$V_{vanes} = 3\text{m/s}$

$V_w = V_v + V_{w/v}$

$10\rightarrow + 4.91\downarrow = 3\downarrow + V_{w/v}$

$\tan\alpha = \frac{4.91 - 3}{10} = .191$



$\alpha = 10.81^{o}$

$V_{w/v} = \frac{10}{\cos\alpha} = 10.18\text{m/s}$

$a_w = a_v + a_{w/v}$

$9.81\downarrow = 0 + a_{w/v}$

$a_{w/v} = 9.81 \text{ m/s}^2\downarrow$

DYNAMICS

PROBLEM N2/11

A car moves around a curve of 270m radius at 80km/h. What is the normal component of acceleration of the car?

$$\rho = 270\text{m}$$

$$V = 80\text{km/h} = 22.2\text{m/sec}$$

$$a = \frac{V^2}{\rho} = \frac{22.2^2}{270}$$

$$a_h = 1.8\text{m/sec}^2$$

$a_n = v^2/r$

$a_t = dv/dt$

PROBLEM N2/12

A car moves around a curve of 270m radius at 80km/h, accelerating at 5km/h/h. What is the total acceleration of the car?

$\rho = 270\text{m}$

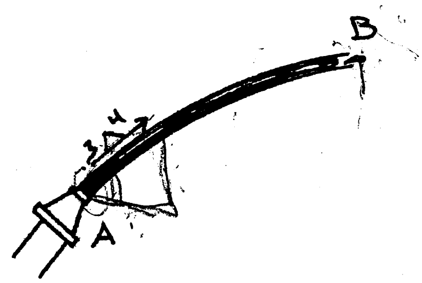
$V = 80\text{km/h} = 22.2\text{m/sec}$

$a_t = 5\text{km/h}^2 = 3.9 \times 10^{-4}\ \text{m/sec}^2$

$$\bar{a} = \frac{dv}{dt}\bar{u}_t + \frac{V^2}{\rho}\bar{u}_n$$

$$\bar{a} = 3.9 \times 10^{-4}\ \bar{u}_t + 1.8\bar{u}_n$$

A nozzle discharges a stream of water in the direction shown with an initial velocity of 25m/s. Determine the radius of curvature of the stream (a) as it leaves the nozzle, (b) at the maximum height of the stream.



a) $a_n = \frac{v^2}{\rho} = \frac{4}{5} g$

$\rho = \frac{5v^2}{4g}$

$\rho = 79.64\text{m}$

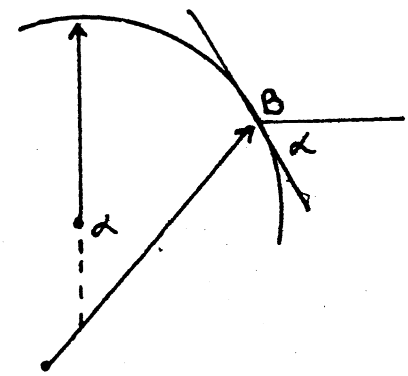
b) $v_B = (v_A)_x = (25)\frac{4}{5} = 20\text{m/s}$

$a_n = g = \frac{v_B^2}{\rho}$

$\rho = \frac{20^2}{9.81}$

$\rho = 40.77\text{m}$

(a) Show that the radius of curvature of the trajectory of a projectile reaches its minimum value at the highest point of the trajectory. (b) Denoting by alpha the angle formed by the trajectory and the horizontal at a given point B, show that the radius of curvature of the trajectory at B is $p = p_{min}/\cos^3\alpha$



$$V = \frac{V_x}{\cos\alpha} \qquad a_n = \frac{V^2}{p} = g\cos\alpha$$

$$p = \frac{\frac{V_x^2}{\cos^2\alpha}}{g\cos\alpha} = \frac{V_x^2}{g\cos^3\alpha}$$

P is min when $\alpha = 0$

$$P_{min} = \frac{V_x^2}{g}$$

$$P_B = \frac{V_x^2}{g\cos^3\alpha} = \frac{P_{min}}{\cos^3\alpha}$$

The motion of particle is defined by $r = 50t^2 - 25t^3$, $\theta = 3t^3$. Find the velocity and acceleration of the particle when t = 2s.

$r = 50t^2 - 25t^3$ $\theta = 3t^3$

$V_r = \frac{dr}{dt} = 100t - 75t^2$ $V_\theta = \frac{d\theta}{dt} = 9t^2$

$a_r = \frac{dv_r}{dt} = 100 - 150t$ $a_\theta = \frac{dV_\theta}{dt} = 18t$

AT t = 2 sec

$$\bar{V} = \frac{dr}{dt}\bar{u}_r + r\frac{d\theta}{dt}\bar{u}_\theta$$

$$= (100t - 75t^2)\bar{u}_r + (50t^2 - 25t^3)(9t^2)\bar{u}\theta$$

$$\bar{V}_2 = (200 - 300)\bar{u}_r + (200 - 200)(36)\bar{u}\theta = -100\bar{u}_r$$

$$\bar{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\bar{u}_r + \left[2\frac{dr}{dt}\frac{d\theta}{dt} + \frac{rd^2\theta}{dt^2}\right]\bar{u}_\theta$$

$$= \left[100 - 150t - (50t^2 - 25t^3)(9t^2\right]^2 \bar{u}_r + \left[2(100t - 75t^2)(9t^2) + (50t^2 - 25t^3)(18t\right]$$

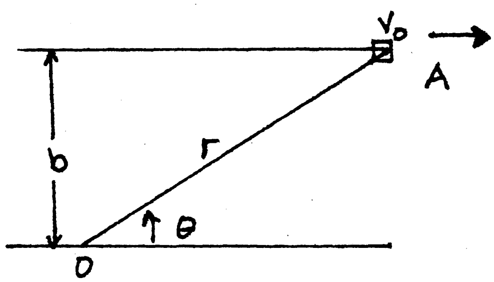
$$= \left[(100 - 300) - (200 - 200)(36)^2\right]\bar{u}_r + \left[2(200 - 300)(36) + (200 - 200)(36)\right]\bar{u}_\theta$$

$$= -200\bar{u}_r - 7200\bar{u}_\theta$$

$$V = \dot{r}\,u_r + r\dot{\theta}\,u_\theta + \dot{z}\,k$$

$$a = (\ddot{r} - r\dot{\theta}^2)u_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})u_\theta + \ddot{z}\,k$$

A wire OA connects the collar A and a reel located at O. Knowing that the collar moves to the right with a constant speed v_o, determine $d\theta/dt$ in terms of v_o, b and θ.



$V_r = \dot{r}$

$V_\theta = r\dot{\theta}$

$$r = \frac{b}{\sin\theta}$$

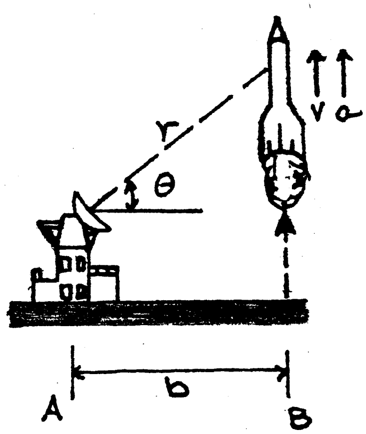
$$V_r = \dot{r} = \frac{-b\cos\theta}{\sin^2\theta}\dot{\theta} \qquad r\dot{\theta} = V_\theta = \frac{b}{\sin\theta}\dot{\theta}$$

$$V^2 = V_o^2 = V_r^2 + V_\theta^2 = \left[\frac{\cos^2\theta}{\sin^4\theta} + \frac{1}{\sin^2\theta}\right] b^2\dot{\theta}^2$$

$$V_o^2 = \left[\frac{\cos^2\theta + \sin^2\theta}{\sin^4\theta}\right] b^2\dot{\theta}^2 = \frac{b^2\dot{\theta}^2}{\sin^4\theta}$$

$$V_o = \pm\frac{b\dot{\theta}}{\sin^2\theta}$$

A rocket is fired vertically from a launching pad at B. Its flight is tracked by radar from point A. Determine the acceleration of the rocket in terms of b, θ, dθ/dt and $d^2\theta/dt^2$.



$$r = b/\cos\theta = b\sec\theta$$

$$V_r = \dot{r} = b\sec\theta\tan\theta\,\dot{\theta}$$

$$V_\theta = r\dot{\theta} = b\sec\theta\,\dot{\theta}$$

$$V = \sqrt{V_r^2 + V_\theta^2}$$

$$= b\sec\theta\sqrt{1 + \tan^2\theta}\quad\dot{\theta}$$

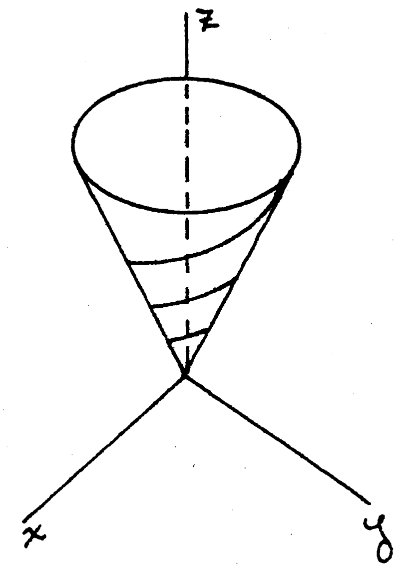
$$\sec^2\theta = 1 + \tan^2\theta$$

$$V = b\sec^2\theta\,\dot{\theta}$$

$$a = \frac{dv}{dt} = 2\,b\sec\theta\,(\sec\theta\tan\theta)\,\dot{\theta}^2 + b\sec^2\theta\,\ddot{\theta}$$

$$a = b\sec^2\theta\left[\ddot{\theta} + 2\dot{\theta}^2\tan\theta\right]$$

The motion of particle on the surface of a right circular cone is defined by the relations $R = ht \tan\beta$, $\theta = 2\pi t$ and $z = ht$ wher beta is the apex angle of the cone and h is the distance the particle rises in one passage around the cone. Determine the magnitude of the velocity and acceleration at any time t.



$z = ht$ $\qquad R = ht\tan\beta$ $\qquad \theta = 2\pi t$

$\dot{z} = h$ $\qquad \dot{R} = h\tan\beta$ $\qquad \dot{\theta} = 2\pi$

$\ddot{z} = 0$ $\qquad \ddot{R} = 0$ $\qquad \ddot{\theta} = 0$

$$\bar{V} = \dot{R}\bar{u}_R + R\dot{\theta}\bar{u}_\theta + \dot{z}k$$

$$= h\tan\beta\,\bar{u}_R + (ht\tan\beta)(2\pi)\bar{u}_\theta + h\bar{k}$$

$$V = h\sqrt{\tan^2\beta + 4\pi^2t^2\tan^2\beta + 1}$$

$$V = h\sqrt{4\pi^2t^2\tan^2\beta + \sec^2\beta}$$

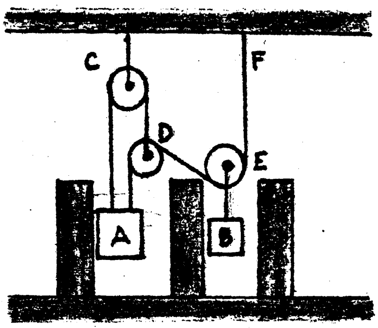
$$\bar{a} = (\ddot{R} - R\dot{\theta}^2)\bar{u}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\bar{u}_\theta + \ddot{z}\bar{k}$$

$$= (ht\tan\beta\, 4\pi^2)\bar{u}_R + (2h\tan\beta\,(2\pi)\bar{u}_\theta + 0\bar{k}$$

$$a^2 = (4\pi^2 ht\tan\beta)^2 + (4\pi h\tan\beta)^2$$

$$a = 4\pi h\tan\beta\sqrt{1 + \pi^2t^2}$$

Knowing that block B moves downward with a constant velocity of 180mm/s determine (a) the velocity of block A, (b) the velocity of pulley D.



CABLE ACD

$X_A + X_D = \text{const.}$

$V_A + V_D = 0 \qquad V_A = -V_D$

CABLE ADEF

$(X_A - X_D) + (X_E - X_D) + X_E = \text{const.}$

$X_A - 2X_D + 2X_E = \text{const.}$

$V_A - 2V_D + 2V_E = 0$

$V_A = -V_D$

$-V_D - 2V_D + 2V_E = 0$

$-3V_D + 2V_E = 0$

$V_B = V_E = 180\text{mm/sec}$

$-3V_D + 2(180) = 0$

$V_D = 120 \text{ mm/sec} \downarrow$

$V_A = -V_D$

$V_A = 120\text{m/sec} \uparrow$