If the location of a particle is defined by $x = 2t^3 - 3t^2 + t - 1$ where x is in m and t is in sec, find the location, velocity and acceleration when t = 2 sec.

LOCATION at t = 2 sec.

$x = 2(2)^3 - 3(2)^2 + 2 - 1$

$x = 5m$

VELOCITY

$v = \frac{dx}{dt}$

$v = \frac{dx}{dt} = 6t^2 - 6t + 1$

$v = 6(2)^2 - 6(2) + 1$

$v = 13$ m/s

ACCELERATION

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12t - 6$

$a = 12(2) - 6$

$a = 18$ m/s^2

If the positon of a particle is given by $x = 3t^3 - 4t^2 + 7$, find the time when the velocity is zero and the time when the velocity is a minimum.

$$V = \frac{dx}{dt} = 9t^2 - 8t$$

At $V = 0$

$$9t^2 - 8t = 0$$

$$t(9t - 8) = 0$$

$$t = 0 \quad \text{and} \quad t = \frac{8}{9} = .89\text{sec}$$

VELOCITY IS MINIMUM WHEN $\frac{dv}{dt} = 0$

$$\frac{dv}{dt} = 18t - 8 = 0$$

$$t = \frac{4}{9} = .44\text{sec}$$

If a particle has an acceleration of $-3\ m/sec^2$ and starts with $v_o = 15\ m/sec$ from the positon $x = 30\ m$, find the velocity and position when $t_o = 4$ sec.

$$a = -3m/sec^2$$

$$v_o = 15\ m/sec$$

$$x_o = 30m$$

From the Equation

$$x = x_o + v_o t + \frac{1}{2}at^2,$$

$$x = 30 + 15(4) + \frac{1}{2}(-3)(4^2)$$

$$\underline{\underline{x = 66m}}$$

To find the velocity at t = 4sec the Equation

$V = V_o + at$ is used

$$V = 15 + (-3)(4)$$

$$\underline{\underline{V = 3m/s}}$$

If the acceleration of a particle is given by $a = 3t^2 - 18$ and the initial velocity and position are given by $v_o = 0 m/s$ and $x_o = 50 m$, find (a) the time when the velocity is zero, (b) the position and distance traveled when $t = 5$ sec.

(a) $a = \frac{dv}{dt}$

$$\int_{v_o}^{v} dv = \int_{0}^{t} (3t^2 - 18)dt$$

$$v - v_o = \left[t^3 - 18t\right]$$

$$v_o = 0$$

$$v = t^3 - 18t$$

When the velocity is zero

$$t^3 - 18t = 0$$

$$t(t^2 - 18) = 0$$

$$\therefore t = 0 \quad \text{and} \quad t = \pm\sqrt{18}$$

The velocity is zero at

$t = 0$ and $t = 4.24$ sec

(b) $v = \frac{dx}{dt} = t^3 - 18t$

$$\int_{x_o}^{x} dx = \int_{0}^{t} (t^3 - 18t)dt$$

$$x - x_o = \frac{t^4}{4} - 9t^2$$

$$x_o = 50$$

$$x = 50 + \frac{t^4}{4} - 9t^2$$

At $t = 4.24$, $x = -31$

$$x = 50 + \frac{625}{4}\ 225$$

$$x = -18.75$$

$\therefore$ The distance traveled is

$$d = 50-(-31) + 31-18.75$$

$$= 81 + 12.25$$

$$= 93.25 m$$

The acceleration of a particle is defined by the relation $a = 21 - 12x^2$, where a is expressed in m/sec^2 and x in meters. The particle starts with no initial velocity at the position x = 0. Determine (a) the velocity when x = 1.5m, (b) the position where the velocity is again zero, (c) the positon where the velocity is maximum.

(a) $a = V\frac{dv}{dx}$

$vdv = adx$

$$\int_0^V vdv = \int_0^X (21 - 12x^2)dx$$

$$\frac{v^2}{2} = 21x - 4x^3$$

$$v = \pm\sqrt{42x - 8(x^3)}$$

At x = 1.5

$$v = \pm\sqrt{42(1.5) - 8(1.5^3)}$$

$\underline{v = \pm 6}$ m/sec

(b) $21x - 4x^3 = 0$ At V = 0

$$x(21 - 4x^2) = 0$$

$\underline{x = 0 \quad \text{and} \quad x = 2.3\text{m}}$

Note: the other root x = -2.3m represents a position that cannot be reached when the initial condition is v = 0 at x = 0

(c) Velocity is Maximum where

$$\frac{dv}{dt} = a = 0$$

$$a = 0 = 21 - 12x^2$$

$\underline{x = 1.323\text{m}}$

The acceleration of particle is defined by the relation $a = -0.0125v^2$, where a is the acceleration in m/sec^2 and v is the velocity in m/sec. If the particle is given an initial velocity v_o, find the distance it will travel (a) before its velocity drops to half the initial value, (b) before it comes to rest.

$$a = v\frac{dv}{dx} = -0.0125v^2$$

$$\int_{v_o}^{v} \frac{dv}{v} = -.0125 \int_{o}^{x} dx$$

$$\ln v - \ln v_o = -.0125x$$

$$x = 80 \ln\left(\frac{v_o}{v}\right)$$

(a) For $v = \frac{1}{2} v_o$

$$x = 80 \ln\left(\frac{v_o}{v_{o/2}}\right) = 80 \ln(2)$$

$$x = 55.5\text{m}$$

(b) For $v = 0$

$$x = 80 \ln\left(\frac{v_o}{o}\right) = 80 \ln(\infty) = \infty$$

THE PARTICLE WILL NEVER STOP.

$$x = \infty$$

The position of an oscillation particle is defined by the relation $x = A\sin(pt + \emptyset)$. Denoting the velocity and position coordinate when $t = 0$ by v_o and x_o, respectively, show (a) that $\tan\emptyset = x_o p/v_o$, (b) that the maximum value of the position coordinate is

$$A = \sqrt{X_0^2 + (\frac{V_0}{P})^2}$$

$$x = A\sin(pt + \emptyset)$$

$$v = \frac{dx}{dt} = A\,p\cos(pt + \emptyset)$$

$$a = \frac{d^2x}{dt^2} = -Ap^2\sin(pt + \emptyset)$$

(a) When $t = 0$

$$x_o = A\sin\emptyset \qquad \sin\emptyset = \frac{x_o}{A}$$

$$v_o = Ap\cos\emptyset \qquad \cos\emptyset = \frac{v_o}{Ap}$$

$$\tan\emptyset = \frac{\sin\emptyset}{\cos\emptyset} = \frac{\frac{x_o}{A}}{\frac{v_o}{Ap}} = \frac{x_o p}{v_o}$$

$$\tan\emptyset = \frac{x_o p}{v_o}$$

(b) Since $\sin(pt + \emptyset)$ cannot be greater than one, the maximum value of x is A.

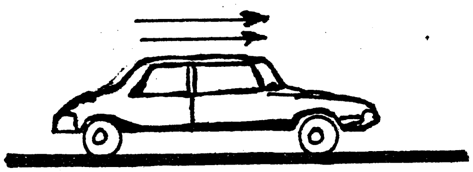
We recall that $\sin^2\emptyset + \cos^2\emptyset = 1$

$$(\frac{x_o}{A})^2 + (\frac{v_o}{Ap})^2 = 1$$

$$x_o^2 + (\frac{v_o}{P})^2 = A^2$$

$$A = \sqrt{x_o^2 + (\frac{v_o}{P})^2}$$

A car travels 300m in 40 sec accelerating at a constant rate of $.2 m/s^2$. Find (a) its initial velocity and final velocity and, (b) the distance traveled during the first half of the time.



(a) $x = x_o + v_o t + \frac{1}{2} a t^2$

$300 = 0 + v_o(40) + \frac{1}{2}(.2)(40)^2$

$v_o = 3.5 \text{ m/s}$

To Find the Final Velocity Use

$V = V_o + at$

$V = 3.5 + (.2)(40)$

$V = 11.5 \text{ m/s}$

(b) $t = 20$ sec

$x = x_o + v_o t + \frac{1}{2} a t^2$

$x = 0 + (3.5)(20) + \frac{1}{2}(.2)(20)^2$

$x = 110\text{m}$

A stone is released from an elevator moving up at a speed of 5m/s and reaches the bottom of the shaft in 3s. (a) How high was the elevator when the stone was released? (b) With what speed does the stone strike the bottom of the shaft:

$a = -9.8 m/sec^2$

(a) $y = y_o + v_o t + \frac{1}{2} at^2$

$0 = y_o + 5(3) + \frac{1}{2}(-9.8)(3)^2$

$y_o = 29.1$

(b) $v_y^2 = v_o^2 + 2a(y - y_o)$

$v_y^2 = 5^2 + 2(-9.8)(0 - 29.1)$

$v_y = 24.4 \downarrow m/sec$

Check

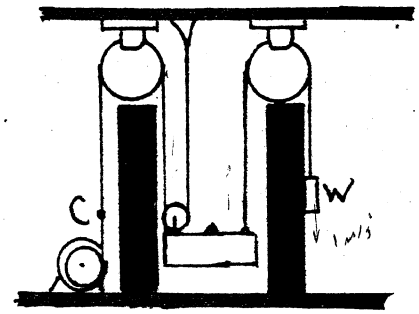
$V = V_o + at$

$V = 5 + (-9.8)(3)$

$V = -24.4$

$V = 24.4 \downarrow$ ✓

The elevator shown starts from rest and moves upward with a constant acceleration. If the counterweight W moves through 8m in 4s, determine (a) the accelerations of the elevator and the cable C, (b) the velocity of the elevator after 4s.



Motion of Counterweight

W:

$$y_W = \frac{1}{2} a_W t^2$$

$$-8 = \frac{1}{2} a_W (4)^2$$

$$a_W = -1$$

$$a_W = 1\text{m/s}^2 \downarrow$$

(a) Since $y_W + y_E = \text{const.}$

$$V_W + V_E = 0 \quad \text{and} \quad a_W + a_E = 0$$

$$\therefore a_E = -a_W = +1$$

$$a_E = 1 \text{ m/s}^2 \uparrow$$

Also $y_c + 2y_E = \text{const.}$

From this

$$v_c + 2v_E = 0 \quad \text{and} \quad a_c + 2a_E = 0$$

$$\therefore a_c = -2a_E = -2(+1) = -2$$

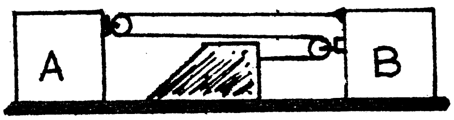
$$a_c = 2\text{m/s}^2 \downarrow$$

(b) Velocity of Elevator

$$V_E = (V_E)_o + a_E t$$

$$V_E = 0 + (+1)(4) = +4 \qquad V_E = 4\text{m/s} \uparrow$$

The slider block B starts from rest and moves to the right with a constant acceleration. After 4s the relative velocity of A with respect to B is 60 mm/s. Determine (a) the accelerations of A and B, (b) the velocity and position of B after 3s.



(a) $3x_B - 2x_A = \text{constant}$

$3v_B - 2v_A = 0$

$v_B = \frac{2}{3} v_A$

$v_{A/B} = v_A - v_B$

$60\text{mm/sec} = v_A - \frac{2}{3} v_A$ at t = 4

$v_A = 180$ mm/sec

$v_B = 120$ mm/sec

$a_A = \frac{v_A}{4} = \frac{180}{4} = 45\ \text{mm/sec}^2$

$a_B = \frac{v_B}{4} = \frac{120}{4} = 30\ \text{mm/sec}^2$

(b) $v_B = (v_B)_o + a_B t$ $(v_B)_o = 0$

$v_B = (30)(3)$

$v_B = 90$ mm/sec

$x_B = x_o^{\nearrow 0} + (v_B)_o t^{\nearrow 0} + \frac{1}{2} a_B t^2$

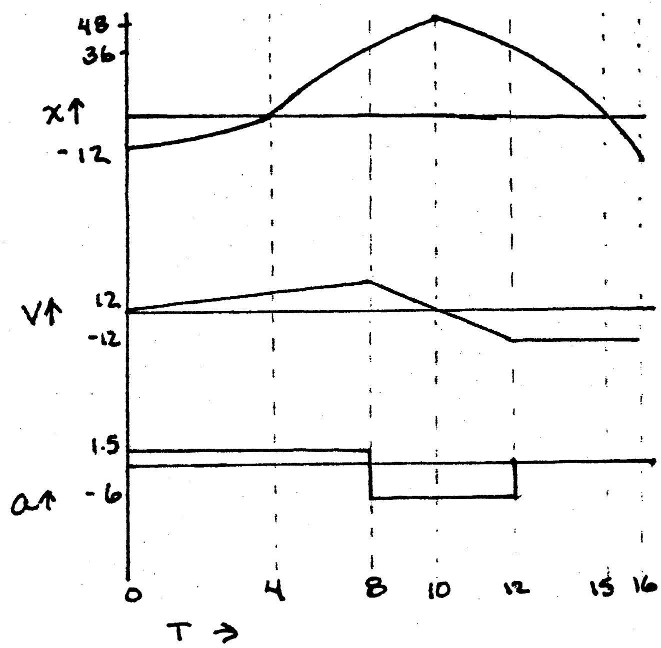
$= (\frac{1}{2})(30)(9)$

$x_B = 135$ mm

DYNAMICS

PROBLEM N1/12

A particle moves in a straight line with the velocity shown in the figure. Knowing that x = -12m at t = 0, draw the a-t and s-t curves for 0<t<16 s and determine (a) the total distance traveled by the particle after 12 s, (b) the two values of t for which the particle passes through the origin.



$x_o = -12$

x-t curve: x = area under v-t curve

$0<t<8 \quad x_B - x_o = \frac{1}{2}(12)(8) = 48 \quad x_B = 36$

$8<t<10 \quad x_{10} - x_B = \frac{1}{2}(12)(2) = 12 \quad x_{10} = 48$

$10<t<12 \quad x_{12} - x_{10} = \frac{1}{2}(-12)(2) = -12 \quad x_{12}=3($

$12<t<16 \quad x_{16} - x_{12} = (-12)(4) = -48 \quad x_{16} = \cdot$

a-t curve: a = slope of v-t curve

$0<t<8 \quad a = 12/8 = 1.5 \text{ m/s}^2$

$8<t<12 \quad a = -24/4 = -6 \text{m/s}^2$

a) 0<t<10 distance = 12 + 48 = 60m

10<t<12 distance = 48 - 36 + 12m

Total distance = 60 + 12 = 72m

b) x = 0 when $t = t_1$ and $t = t_2$ for $t = t_1$, v = 1.5t

t_1: $0 - x_o = \frac{1}{2}(1.5t_1)t_1$

$12 = 1.5/2 \; t_1^2$

$t_1 = 4$ sec.

t_2: $0 - x_{12} = (-12)(t_2 - 12)$

$-36 = -12t_2 + 144$

$t_2 = 15$ sec.

A bus starts from rest at point A and accelerates at the rate of $0.9 m/s^2$ until it reaches a speed of 7.2m/s. It then proceeds at 7.2m/s until the brakes are applied; it comes to rest at point B, 18m beyond the point where the brakes were applie Assuming uniform deceleration and knowing that the distance between A and B is 90m, determine the time required for the bus to travel from A to B.

t_1 = time during uniform acceleration of $.9 m/s^2$

t_2 = time during constant velocity of 7.2m/s

t_3 = time during deceleration.

$$\text{total time} = t_1 + t_2 + t_3$$

x_1 = distance traveled during time t_1

x_2 = distance traveled during time t_2

x_3 = distance traveled during time t_3

$$\text{total distance} = 90 \text{ m}$$
$$x_3 = 18\text{m}$$

TIME t_1

$$v = v_o + at$$
$$7.2 = 0 + (.9)(t_1)$$
$$t_1 = 8\text{s}$$
$$v^2 = v_o^2 + 2a(x_1)$$
$$(7.2)^2 = 0 + 2(.9)(x_1)$$
$$x_1 = 28.8\text{m}$$

TIME t_2

$$x_1 + x_2 + x_3 = 90$$
$$x_2 = 90 - 28.8 - 18$$
$$x_2 = 43.3\text{m}$$
$$x = x_o + v_o t + \tfrac{1}{2}at^2 \quad 0$$
$$x - x_o = v_o t + \cancel{\tfrac{1}{2}at^2}$$
$$43.2 = 7.2(t_2)$$
$$t_2 = 6\text{s}$$

TIME t_3

$$v^2 = v_o^2 + 2a(x - x_o)$$
$$0 = (7.2)^2 + 2a(18)$$
$$a = -1.44 \text{ m/s}^2$$
$$v = v_o + at$$
$$0 = 7.2 + (-1.44)(t_3)$$
$$t_3 = 5 \text{ sec}$$

$$t_T = \text{total time} = 8 + 6 + 5 = 19\text{s}$$

$$\underline{\underline{t_T = 19\text{s}}}$$

DYNAMICS

PROBLEM N1/14

The acceleration record shown was obtained for a truck travelling on a straight highway. Knowing that the initial velocity of the truck was 18 km/h, determine the velocity and distance traveled when (a) t = 4s, (b) t = 6s.

PROCEDURE

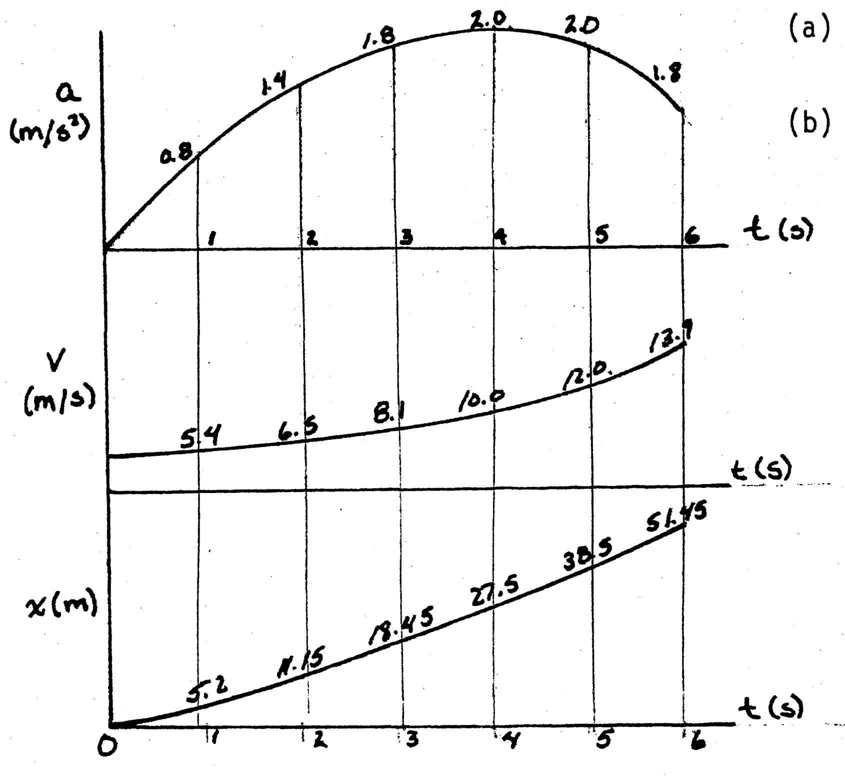
1. We approximate the a-t curve by rectangles as shown below. For each interval of time t = 1.0 sec, we have change in v = area under a-t curve. Δv = (a average)Δt

2. Noting that v_0 = 18 km/h = 5 m/s, we construct v-t curve by successively adding the changes in velocity Δv: v_0 = 5m/s

3. We now approximate the v-t curve to determine the area under the curve.
   Δx = area under v-t curve
   Δx = (v average)Δt

4. Noting that x_0 = 0, we construct the x-t curve by successively adding the changes in positon Δx

Δt = 1.0 sec



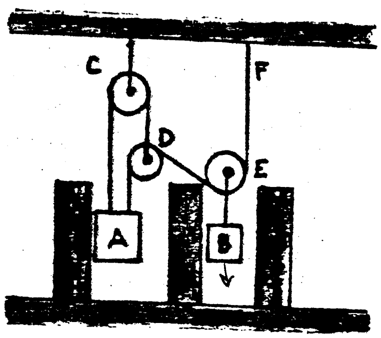
(a) t = 4s: v = 10 m/s x = 27.5m

(b) t = 6s: v = 13.9 m/s x = 51.5m.

DYNAMICS

PROBLEM N 1/15

Knowing that block B moves downward with a constant velocity of 180mm/s determine (a) the velocity of block A, (b) the velocity of pulley D.



CABLE ACD

$X_A + X_D = \text{const.}$

$V_A + V_D = 0 \qquad V_A = -V_D$

CABLE ADEF

$(X_A - X_D) + (X_E - X_D) + X_E = \text{const.}$

$X_A - 2X_D + 2X_E = \text{const.}$

$V_A - 2V_D + 2V_E = 0$

$V_A = -V_D$

$-V_D - 2V_D + 2V_E = 0$

$-3V_D + 2V_E = 0$

$V_B = V_E = 180\text{mm/sec}$

$-3V_D + 2(180) = 0$

$V_D = 120 \text{ mm/sec} \downarrow$

$V_A = -V_D$

$V_A = 120\text{m/sec} \uparrow$